

10. ESPRESSIONI “TIPO MONOMI, POLINOMI, PRODOTTI NOTEVOLI”

Osservazione preliminare, ovvia ma fondamentale:

♥ **quando una radice quadrata è elevata al quadrato, la radice scompare e rimane solo il radicando.**

♥ **Lo stesso avviene se una radice quadrata è moltiplicata per sé stessa.**

$$\boxed{(\sqrt{7})^2 = 7; \quad \sqrt{7} \cdot \sqrt{7} = \begin{cases} (\sqrt{7})^2 = 7 \\ \sqrt{7 \cdot 7} = \sqrt{7^2} = 7 \end{cases}}$$

Ed ecco qualche esempio svolto.

- a) $(3\sqrt{5})^2 = 3^2 \cdot (\sqrt{5})^2 = 9 \cdot 5 = 45$
- b) $\sqrt{3}(\sqrt{5} + \sqrt{3} - 2\sqrt{6}) = \sqrt{15} + 3 - 2\sqrt{18} = \sqrt{15} + 3 - 2 \cdot 3\sqrt{2} = \sqrt{15} + 3 - 6\sqrt{2}$
- c) $(a\sqrt{b} - \sqrt[3]{b})(\sqrt{ab} - b\sqrt{b}) = a\sqrt{ab^2} - ab \cdot b - \sqrt[3]{b}\sqrt{ab} + b\sqrt[3]{b}\sqrt{b} =$
 $= ab\sqrt{a} - ab^2 - \sqrt[6]{b^2} \sqrt[6]{a^3 b^3} + b\sqrt[6]{b^2} \sqrt[6]{b^3} = ab\sqrt{a} - ab^2 - \sqrt[6]{a^3 b^5} + b\sqrt[6]{b^5}$
- d) $(5\sqrt{2} - 2\sqrt{3})^2 = (5\sqrt{2})^2 + 2 \cdot 5\sqrt{2} \cdot (-2\sqrt{3}) + (-2\sqrt{3})^2 = 50 - 20\sqrt{6} + 12 = 62 - 20\sqrt{6} = 2(31 - 10\sqrt{6})$
- e) $(\sqrt[4]{a} + 3)^2 = \left(\sqrt[2]{\sqrt[4]{a}}\right)^2 + 2 \cdot \sqrt[4]{a} \cdot 3 + 3^2 = \sqrt{a} + 6\sqrt[4]{a} + 9$
- f) $(2\sqrt{2} + \sqrt{7})(2\sqrt{2} - \sqrt{7}) = (2\sqrt{2})^2 - (\sqrt{7})^2 = 8 - 7 = 1$
- g) $(\sqrt{a-1} + \sqrt{a+1})(\sqrt{a-1} - \sqrt{a+1}) = (\sqrt{a-1})^2 - (\sqrt{a+1})^2 = (a-1) - (a+1) = \cancel{a} - 1 - \cancel{a} - 1 = -2$
- h) $(\sqrt{a-1} - \sqrt{a+1})^2 = (\sqrt{a-1})^2 + (-\sqrt{a+1})^2 + 2\sqrt{a-1}(-\sqrt{a+1}) =$
 $= a - 1 + a + 1 - 2\sqrt{(a-1)(a+1)} = 2a - 2\sqrt{a^2 - 1} = 2(a - \sqrt{a^2 - 1})$
- i) $(\sqrt{\sqrt{x} + \sqrt{y}} + \sqrt{\sqrt{x} - \sqrt{y}})^2 = (\sqrt{\sqrt{x} + \sqrt{y}})^2 + (\sqrt{\sqrt{x} - \sqrt{y}})^2 + 2\sqrt{\sqrt{x} + \sqrt{y}} \cdot \sqrt{\sqrt{x} - \sqrt{y}} =$
 $= \sqrt{x} + \cancel{\sqrt{y}} + \sqrt{x} - \cancel{\sqrt{y}} + 2\sqrt{(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})} = 2\sqrt{x} + 2\sqrt{x - y} = 2(\sqrt{x} + \sqrt{x - y})$
- j) $(5 - 2\sqrt{2} - \sqrt{3})^2 = 5^2 + (-2\sqrt{2})^2 + (-\sqrt{3})^2 + 2 \cdot 5 \cdot (-2\sqrt{2}) + 2 \cdot 5 \cdot (-\sqrt{3}) + 2 \cdot (-2\sqrt{2}) \cdot (-\sqrt{3}) =$
 $= 25 + 8 + 3 - 20\sqrt{2} - 10\sqrt{3} + 4\sqrt{6} = 36 - 20\sqrt{2} - 10\sqrt{3} + 4\sqrt{6} = 2(18 - 10\sqrt{2} - 5\sqrt{3} + 2\sqrt{6})$
- k) $(\sqrt[3]{a} + \sqrt[3]{b})^3 = (\sqrt[3]{a})^3 + 3 \cdot (\sqrt[3]{a})^2 \cdot \sqrt[3]{b} + 3 \cdot \sqrt[3]{a} \cdot (\sqrt[3]{b})^2 + (\sqrt[3]{b})^3 = a + 3\sqrt[3]{a^2 b} + 3\sqrt[3]{a b^2} + b$
- l) $(2\sqrt{2} - 1)^3 = (2\sqrt{2})^3 + 3 \cdot (2\sqrt{2})^2 \cdot (-1) + 3 \cdot 2\sqrt{2} \cdot (-1)^2 + (-1)^3 =$
 $= 8\sqrt{8} + 3 \cdot 8 \cdot (-1) + 3 \cdot 2\sqrt{2} \cdot 1 - 1 = 16\sqrt{2} - 24 + 6\sqrt{2} - 1 = 22\sqrt{2} - 25$

Nella pratica del calcolo, si possono tracciare delle “**barre di semplificazione**” (noi, negli esercizi svolti di questo testo, **non sempre le abbiamo riportate** sia per ragioni grafiche che per lasciare questo compito, se lo ritiene, al lettore):

$$\begin{aligned} (\sqrt{7})^2 &= 7 & \sqrt{7^2} &= 7 \\ \sqrt{x} \cdot \sqrt{x^{14}} &= \sqrt{x^{15}} = x^3 \\ (-5 \sqrt[6]{3})^2 &= (-5)^2 (\sqrt[6]{3})^2 = 25 \sqrt[3]{3} \end{aligned}$$

Fai ora tu qualcuno di questi ESERCIZI:

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| 1) $(2\sqrt{3})^2$ | 2) $(3\sqrt{a+b})^2$ | 3) $(-2\sqrt{2})^2$ |
| 4) $(\sqrt{2}+1)(\sqrt{2}-1)$ | 5) $(\sqrt{3}+2)(\sqrt{3}-2)$ | 6) $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ |
| 7) $(2\sqrt{3}+1)(2\sqrt{3}-1)$ | 8) $(2\sqrt{2}+3)(2\sqrt{2}-3)$ | 9) $(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})$ |
| 10) $(\sqrt{2}+1)^2$ | 11) $(\sqrt{3}-\sqrt{2})^2$ | 12) $(2\sqrt{2}+3)^2$ |
| 13) $(2\sqrt{5}-3\sqrt{2})^2$ | 14) $(\sqrt{3}-\sqrt{2}+1)^2$ | 15) $(2\sqrt{7}+3\sqrt{5}+4\sqrt{3})^2$ |
| 16) $\sqrt{3}(\sqrt{3}+2\sqrt{2})$ | 17) $(\sqrt{2}+2)(\sqrt{2}-1)$ | 18) $(2\sqrt{3}+\sqrt{2})(\sqrt{3}-2\sqrt{2})$ |
| 19) $(1+2\sqrt{7})(3+4\sqrt{11})$ | 20) $2\sqrt{5}(2\sqrt{10}-3\sqrt{5})$ | 21) $(\sqrt{3}+\sqrt{2}+1)(\sqrt{3}-\sqrt{2}+1)$ |
| 22) $(\sqrt[4]{2}+1)(\sqrt[4]{2}-1)$ | 23) $(\sqrt[4]{2}-1)^2$ | 24) $(\sqrt[4]{2}+\sqrt[3]{2})(\sqrt[4]{2}-\sqrt[3]{2})$ |
| 25) $(\sqrt[4]{2}+\sqrt[3]{2})^2$ | 26) $(\sqrt{3}+1)^3$ | 27) $(\sqrt{3}+1)^4$ |
| 28) $(2\sqrt{3}-3)^3$ | 29) $(\sqrt{3}+2\sqrt{2})^3$ | 30) $(\sqrt{2}-1)^5$ |
| 31) $(\sqrt{3}-\sqrt{2})^6$ | 32) $(a\sqrt{b}+b\sqrt{a})^2$ | 33) $(\sqrt{a}-\sqrt{b})^3$ |
| 34) $(\sqrt{x+2}+\sqrt{x+1})(\sqrt{x+2}-\sqrt{x+1})$ | 35) $(\sqrt{a+b}+\sqrt{a-b})^2$ | |
| 36) $(\sqrt[4]{a}-\sqrt[4]{b})(\sqrt[4]{a}+\sqrt[4]{b})(\sqrt{a}+\sqrt{b})$ | 37) $(2\sqrt{a}-\sqrt{3a-1})(2\sqrt{a}+\sqrt{3a-1})$ | |
| 38) $(\sqrt{x+2}+\sqrt{x+1})^2$ | 39) $(2\sqrt{a}-\sqrt{b}-1)^2$ | |
| 40) $\sqrt[4]{a}(\sqrt[4]{a}+\sqrt[5]{a})$ | 41) $(\sqrt{x}-\sqrt[4]{x}+1)^2$ | |
| 42) $\sqrt{a}(\sqrt{a}+\sqrt{b})$ | 43) $(\sqrt{x}+\sqrt{y}-1)(\sqrt{x}-\sqrt{y}+1)$ | |
| 44) $(3\sqrt{x}+4)(5\sqrt{x}+6)$ | 45) $(x\sqrt{x}+1)(x\sqrt{x}-2)$ | |
| 46) $3\sqrt{x}(2\sqrt{x}-5y)$ | 47) $(\sqrt{1+\sqrt{x}}+\sqrt{1-\sqrt{x}})^2$ | |
| 48) $(\sqrt{1+\sqrt{x}}+\sqrt{1-\sqrt{x}})(\sqrt{1+\sqrt{x}}-\sqrt{1-\sqrt{x}})$ | 49) $(\sqrt{a}-\sqrt{a+b})^2$ | ALTRI ESERCIZI a pag. 23 |

RISULTATI

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|---|-------------------------------------|---|--|---------------------------------------|------|-------|-------|------|
| 1) 12 | 2) $9(a+b)=9a+9b$ | 3) 8 | 4) 1 | 5) -1 | 6) 1 | 7) 11 | 8) -1 | 9) 2 |
| 10) $3+2\sqrt{2}$ | 11) $5-2\sqrt{6}$ | 12) $17+12\sqrt{2}$ | 13) $38-12\sqrt{10}$ | 14) $6-2\sqrt{6}+2\sqrt{3}-2\sqrt{2}$ | | | | |
| 15) $121+12\sqrt{35}+16\sqrt{21}+24\sqrt{15}$ | 16) $3+2\sqrt{6}$ | 17) $\sqrt{2}$ | 18) $2-3\sqrt{6}$ | | | | | |
| 19) $3+4\sqrt{11}+6\sqrt{7}+8\sqrt{77}$ | 20) $20\sqrt{2}-30=10(2\sqrt{2}-3)$ | 21) $2+2\sqrt{3}=2(1+\sqrt{3})$ | | | | | | |
| 22) $\sqrt{2}-1$ | 23) $\sqrt{2}+1-2\sqrt[4]{2}$ | 24) $\sqrt{2}-\sqrt[3]{4}$ | 25) $\sqrt{2}+\sqrt[3]{4}+2\sqrt[12]{128}$ | 26) $6\sqrt{3}+10$ | | | | |
| 27) $28+16\sqrt{3}$ | 28) $78\sqrt{3}-135$ | 29) $27\sqrt{3}+34\sqrt{2}$ | 30) $29\sqrt{2}-41$ | 31) $485-198\sqrt{6}$ | | | | |
| 32) $a^2b+ab^2+2ab\sqrt{ab}$ | 33) $(a+3b)\sqrt{a}-(3a+b)\sqrt{b}$ | 34) 1 | 35) $2(a+\sqrt{a^2-b^2})$ | 36) $a-b$ | | | | |
| 37) $a+1$ | 38) $2x+3+2\sqrt{x^2+3x+2}$ | 39) $4a+b+1-4\sqrt{ab}-4\sqrt{a}+2\sqrt{b}$ | 40) $\sqrt{a}+\sqrt[20]{a^9}$ | | | | | |
| 41) $x+1+3\sqrt{x}-2\sqrt[4]{x^3}-2\sqrt[4]{x}$ | 42) $a+\sqrt{ab}$ | 43) $x-y-1+2\sqrt{y}$ | 44) $15x+38\sqrt{x}+24$ | | | | | |
| 45) $x^3-x\sqrt{x}-2$ | 46) $6x-15y\sqrt{x}$ | 47) $2(1+\sqrt{1-x})$ | 48) $2\sqrt{x}$ | 49) $2a+b-2\sqrt{a^2+ab}$ | | | | |