SISTEMI DI GRADO SUPERIORE AL 1° CON TRE O PIU' INCOGNITE
Correzione degli esercizi "dispari"

3)
$$\begin{cases} x+y+t=2 \\ x-t=2 \\ x^2+t^2=y^2+6 \end{cases}$$
$$\begin{cases} x=t+2 \\ t\cancel{+2}+y+t=\cancel{2};\quad y=-2t \\ x^2+t^2=y^2+6 \end{cases}$$
$$\begin{cases} x=t+2 \\ y=-2t \\ (t+2)^2+t^2=(-2t)^2+6 \end{cases}$$
$$t^2+4t+4+t^2=4t^2+6$$
$$2t^2-4t+2=0$$
$$t^2-2t+1=0$$
$$(t-1)^2=0$$
$$t=1$$
$$\begin{cases} t=1 \\ x=t+2=1+2=3 \\ y=-2t=-2\cdot 1=-2 \end{cases}$$
$$\begin{cases} x=3 \\ y=-2 \\ t=1 \end{cases}$$

5)
$$\begin{cases} x-2y=0 \\ y-2z=0 \\ xyz=8 \end{cases}$$
$$\begin{cases} x=2y \\ -2z=-y;\quad z=\dfrac{y}{2} \\ 2y\cdot y\cdot \dfrac{y}{2}=8 \end{cases}$$
$$\begin{cases} x=2y \\ z=\dfrac{y}{2} \\ y^3=8;\ \ y=2 \end{cases}$$
$$\begin{cases} y=2 \\ x=2y=4 \\ z=\dfrac{y}{2}=1 \end{cases}$$
$$\begin{cases} x=4 \\ y=2 \\ z=1 \end{cases}$$

7)

$$\begin{cases} 2x+y=10 \\ 0{,}3y-0{,}2z=\dfrac{2}{5} \\ xy+z^2=9 \end{cases}$$

$$\begin{cases} 2x+y=10 \\ 3y-2z=4 \quad (\textit{ottenuta tramite moltiplicazione per } 10) \\ xy+z^2=9 \end{cases}$$

La prima equazione contiene solo x e y, la seconda solo y e z; converrà quindi esprimere sia x che z in funzione di y:

$$\begin{cases} x=\dfrac{10-y}{2} \\ z=\dfrac{3y-4}{2} \\ \dfrac{10-y}{2}\cdot y+\left(\dfrac{3y-4}{2}\right)^2=9 \end{cases}$$

$$\frac{10y-y^2}{2}+\frac{9y^2-24y+16}{4}=9$$

$$20y-2y^2+9y^2-24y+16=36$$

$$7y^2-4y-20=0$$

$$y=\frac{2\pm\sqrt{4+140}}{7}=\frac{2\pm\sqrt{144}}{7}=\frac{2\pm12}{7}=\left\langle\begin{matrix} -\dfrac{10}{7} \\ 2 \end{matrix}\right.$$

$$\begin{cases} x=\dfrac{10-y}{2}=\dfrac{10+\dfrac{10}{7}}{2}=\dfrac{\dfrac{80}{7}}{2}=\dfrac{40}{7} \\ y=-\dfrac{10}{7} \\ z=\dfrac{3y-4}{2}=\dfrac{3\left(-\dfrac{10}{7}\right)-4}{2}=\dfrac{-\dfrac{30}{7}-4}{2}=\dfrac{-\dfrac{58}{7}}{2}=-\dfrac{29}{7} \end{cases}$$

$$\begin{cases} x=\dfrac{10-y}{2}=\dfrac{10-2}{2}=\dfrac{8}{2}=4 \\ y=2 \\ z=\dfrac{3y-4}{2}=\dfrac{6-4}{2}=\dfrac{2}{2}=1 \end{cases}$$

9)

$$\begin{cases} x+y+z+t=5 \\ x-y-z+t=1 \\ x-y+z-t=-1 \\ xy+zt=3 \end{cases}$$

Per questo sistema in 4 incognite,
risulta conveniente applicare dapprima il metodo di riduzione (=addizione e sottrazione),
in modo da ottenere equazioni molto semplici.

$$\begin{array}{l} (1)+(2) \\ (1)+(3) \\ (2)+(3) \\ \end{array} \begin{cases} 2x+2t=6; \quad x+t=3 \\ 2x+2z=4; \quad x+z=2 \\ 2x-2y=0; \quad x-y=0; \quad x=y \\ xy+zt=3 \end{cases}$$

A questo punto, è possibile servirsi delle prime 3 equazioni per esprimere y, z, t in funzione di x:

$$\begin{cases} t=3-x \\ z=2-x \\ y=x \\ x^2+(2-x)(3-x)=3 \end{cases}$$

$$x^2+6-2x-3x+x^2=3$$

$$2x^2-5x+3=0$$

$$x=\frac{5\pm\sqrt{25-24}}{4}=\frac{5\pm 1}{4}=\left\langle \begin{array}{l} 1 \\ \frac{3}{2} \end{array} \right.$$

$$\begin{cases} x=1 \\ y=x=1 \\ z=2-x=2-1=1 \\ t=3-x=3-1=2 \end{cases} \qquad \begin{cases} x=\frac{3}{2} \\ y=x=\frac{3}{2} \\ z=2-x=2-\frac{3}{2}=\frac{1}{2} \\ t=3-x=3-\frac{3}{2}=\frac{3}{2} \end{cases}$$

11)

$$\begin{cases} \frac{x}{6}+\frac{y}{3}+\frac{z}{2}-\frac{4}{3}=0 \\ 2x+y+6z=13 \\ x^2+yz=2 \end{cases}$$

$$\begin{cases} x+2y+3z=8 \ \ (\textit{moltiplicazione per } 6) \\ 2x+y+6z=13 \\ x^2+yz=2 \end{cases}$$

Caso fortunato:
se si moltiplica la prima equazione per 2 e le si sottrae la seconda,
si può ricavare immediatamente l'incognita y!

$$2\cdot\begin{cases} x+2y+3z=8 \\ 2x+y+6z=13 \\ x^2+yz=2 \end{cases}$$

$$\begin{cases} 2x+4y+6z=16 \\ 2x+y+6z=13 \\ x^2+yz=2 \end{cases}$$

$$\begin{matrix} (1)-(2) \\ (2) \\ (3) \end{matrix}\begin{cases} 3y=3; \quad y=1 \\ 2x+y+6z=13 \\ x^2+yz=2 \end{cases}$$

$$\begin{cases} y=1 \\ 2x+1+6z=13; \quad 2x+6z=12; \quad x+3z=6 \\ x^2+z=2 \end{cases}$$

$$\begin{cases} y=1 \\ z=2-x^2 \\ x+3\left(2-x^2\right)=6; \ \ x\cancel{+6}-3x^2=\cancel{6}; \ \ 3x^2-x=0; \quad x\left(3x-1\right)=0; \quad x=0 \vee x=1/3 \end{cases}$$

$$\begin{cases} x=0 \\ y=1 \\ z=2-x^2=2 \end{cases} \qquad \begin{cases} x=\frac{1}{3} \\ y=1 \\ z=2-x^2=2-\frac{1}{9}=\frac{17}{9} \end{cases}$$

13)

$$\begin{cases} x^2+y^2=\dfrac{25}{9} \\ 5z-x-y=\dfrac{5}{3} \\ xy=\dfrac{5}{3}z \end{cases}$$

$$\begin{cases} (x+y)^2-2xy=\dfrac{25}{9} \\ x+y=5z-\dfrac{5}{3} \\ xy=\dfrac{5}{3}z \end{cases}$$

$$\begin{cases} x+y=5z-\dfrac{5}{3} \\ xy=\dfrac{5}{3}z \\ \left(5z-\dfrac{5}{3}\right)^2-2\cdot\dfrac{5}{3}z=\dfrac{25}{9}; \quad 25z^2-\dfrac{50}{3}z+\cancel{\dfrac{25}{9}}-\dfrac{10}{3}z=\cancel{\dfrac{25}{9}} \end{cases}$$

$$25z^2-\frac{\cancel{60}^{20}}{\cancel{3}}z=0; \quad 5z^2-4z=0; \quad z(5z-4)=0; \quad z=0 \ \vee \ z=\frac{4}{5}$$

$$\begin{cases} z=0 \\ x+y=5z-\dfrac{5}{3}=-\dfrac{5}{3} \\ xy=\dfrac{5}{3}z=0 \end{cases}$$

$$t^2+\frac{5}{3}t=0$$

$$t\left(t+\frac{5}{3}\right)=0$$

$$t=0 \ \vee \ t=-\frac{5}{3}$$

$$\boxed{\begin{cases} x=0 \\ y=-5/3 \\ z=0 \end{cases}} \quad \boxed{\begin{cases} x=-5/3 \\ y=0 \\ z=0 \end{cases}}$$

$$\begin{cases} z=\dfrac{4}{5} \\ x+y=5z-\dfrac{5}{3}=5\cdot\dfrac{4}{5}-\dfrac{5}{3}=4-\dfrac{5}{3}=\dfrac{7}{3} \\ xy=\dfrac{5}{3}z=\dfrac{5}{3}\cdot\dfrac{4}{5}=\dfrac{4}{3} \end{cases}$$

$$t^2-\frac{7}{3}t+\frac{4}{3}=0$$

$$3t^2-7t+4=0$$

$$3t^2-3t-4t+4=0$$

$$3t(t-1)-4(t-1)=0$$

$$(t-1)(3t-4)=0$$

$$t=1 \ \vee \ t=4/3$$

$$\boxed{\begin{cases} x=1 \\ y=4/3 \\ z=4/5 \end{cases}} \quad \boxed{\begin{cases} x=4/3 \\ y=1 \\ z=4/5 \end{cases}}$$