SISTEMI SIMMETRICI

Correzione degli esercizi n. 5, 9, 11, 13, 15, 16

5)

$$\begin{cases} x+y=\dfrac{1}{6} \\ x^2+y^2=\dfrac{13}{36} \end{cases}$$

$$\begin{cases} x+y=\dfrac{1}{6} \\ (x+y)^2-2xy=\dfrac{13}{36} \end{cases} \qquad \begin{cases} x+y=\dfrac{1}{6} \\ \dfrac{1}{36}-2xy=\dfrac{13}{36}; \quad -2xy=\dfrac{\cancel{12}^{\,1}}{\cancel{36}_{\,3}}; \quad xy=-\dfrac{1}{6} \end{cases}$$

$$t^2-\frac{1}{6}t-\frac{1}{6}=0$$

$$6t^2-t-1=0$$

$$t_{1,2}=\frac{1\pm\sqrt{1+24}}{12}=\frac{1\pm 5}{12}=\left\langle \begin{array}{l} -\dfrac{4}{12}=-\dfrac{1}{3} \\ \dfrac{6}{12}=\dfrac{1}{2} \end{array}\right.$$

$$\begin{cases} x=-\dfrac{1}{3} \\ y=\dfrac{1}{2} \end{cases} \quad \vee \quad \begin{cases} x=\dfrac{1}{2} \\ y=-\dfrac{1}{3} \end{cases}$$

9)

$$\begin{cases} xy=16 \\ x^2+y^2=32 \end{cases}$$

$$\begin{cases} xy=16 \\ (x+y)^2-2xy=32 \end{cases}$$

$$\begin{cases} xy=16 \\ (x+y)^2-32=32; \quad (x+y)^2=64; \quad x+y=\pm 8 \end{cases}$$

$$\begin{cases} xy=16 \\ x+y=8 \end{cases} \quad \vee \quad \begin{cases} xy=16 \\ x+y=-8 \end{cases}$$

$t^2-8t+16=0$ — $t^2+8t+16=0$

$(t-4)^2=0$ — $(t+4)^2=0$

$t_{1,2}=4$ — $t_{1,2}=-4$

$$\begin{cases} x=4 \\ y=4 \end{cases} \textit{soluz. "doppia"} \qquad \begin{cases} x=-4 \\ y=-4 \end{cases} \textit{soluz. "doppia"}$$

11)

$$\begin{cases} xy = 3 \\ x^2 + y^2 = 14 \end{cases} \quad \begin{cases} xy = 3 \\ (x+y)^2 - 2xy = 14 \end{cases}$$

$$\begin{cases} xy = 3 \\ (x+y)^2 - 6 = 14; \quad (x+y)^2 = 20; \quad x + y = \pm\sqrt{20} = \pm 2\sqrt{5} \end{cases}$$

$$\begin{cases} x + y = 2\sqrt{5} \\ xy = 3 \end{cases} \qquad \begin{cases} x + y = -2\sqrt{5} \\ xy = 3 \end{cases}$$

$$t^2 - 2t\sqrt{5} + 3 = 0 \qquad t^2 + 2t\sqrt{5} + 3 = 0$$

$$t_{1,2} = \sqrt{5} \pm \sqrt{5-3} = \sqrt{5} \pm \sqrt{2} \qquad t_{1,2} = -\sqrt{5} \pm \sqrt{5-3} = -\sqrt{5} \pm \sqrt{2}$$

$$\begin{cases} x = \sqrt{5} + \sqrt{2} \\ y = \sqrt{5} - \sqrt{2} \end{cases} \quad \begin{cases} x = \sqrt{5} - \sqrt{2} \\ y = \sqrt{5} + \sqrt{2} \end{cases} \quad \begin{cases} x = -\sqrt{5} - \sqrt{2} \\ y = -\sqrt{5} + \sqrt{2} \end{cases} \quad \begin{cases} x = -\sqrt{5} + \sqrt{2} \\ y = -\sqrt{5} - \sqrt{2} \end{cases}$$

13)

$$\begin{cases} x + y = 2a \\ x^2 + y^2 = 2\left(a^2 + b^2\right) \end{cases}$$

$$\begin{cases} x + y = 2a \\ (x+y)^2 - 2xy = 2\left(a^2 + b^2\right); \end{cases}$$

$$4a^2 - 2xy = 2a^2 + 2b^2; \quad -2xy = -2a^2 + 2b^2;$$

$$2xy = 2a^2 - 2b^2; \quad xy = a^2 - b^2$$

$$\begin{cases} x + y = 2a \\ xy = a^2 - b^2 \end{cases}$$

$$t^2 - 2at + \left(a^2 - b^2\right) = 0$$

$$t = a \pm \sqrt{a^2 - \left(a^2 - b^2\right)} = a \pm \sqrt{\cancel{a^2} - \cancel{a^2} + b^2} = a \pm b$$

$$\begin{cases} x = a + b \\ y = a - b \end{cases} \vee \begin{cases} x = a - b \\ y = a + b \end{cases}$$

15)

$$\begin{cases} x^3+y^3=14 \\ x+y=2 \end{cases}$$

$$\boxed{x^3+y^3=(x+y)^3-3x^2y-3xy^2=(x+y)^3-3xy(x+y)}$$

$$\begin{cases} (x+y)^3-3xy(x+y)=14 \\ x+y=2 \end{cases}$$

$$\begin{cases} 8-6xy=14; \quad -6xy=6; \quad xy=-1 \\ x+y=2 \end{cases}$$

$$t^2-2t-1=0$$

$$t_{1,2}=1\pm\sqrt{1+1}=1\pm\sqrt{2}$$

$$\begin{cases} x=1-\sqrt{2} \\ y=1+\sqrt{2} \end{cases} \vee \begin{cases} x=1+\sqrt{2} \\ y=1-\sqrt{2} \end{cases}$$

16)

$$\begin{cases} x^3+y^3=\frac{1}{4} \\ x+y=1 \end{cases}$$

$$\begin{cases} (x+y)^3-3xy(x+y)=\frac{1}{4} \\ x+y=1 \end{cases}$$

$$\begin{cases} 1-3xy=\frac{1}{4}; \quad 3xy=1-\frac{1}{4}; \quad 3xy=\frac{3}{4}; \quad xy=\frac{1}{4} \\ x+y=1 \end{cases}$$

$$t^2-t+\frac{1}{4}=0$$

$$4t^2-4t+1=0$$

$$(2t-1)^2=0$$

$$t_{1,2}=\frac{1}{2}$$

$$\begin{cases} x=\frac{1}{2} \\ y=\frac{1}{2} \end{cases}$$