SISTEMI DI GRADO SUPERIORE AL 1°
Correzione di alcuni degli esercizi proposti:
n. 3, 4, 9, 12, 14

3)

$$\begin{cases} 3x - y = 8 \\ x^2 + y^2 = 10 \end{cases}$$

$$\begin{cases} -y = 8 - 3x; \quad y = 3x - 8 \\ x^2 + (3x-8)^2 = 10 \end{cases}$$

$$x^2 + 9x^2 - 48x + 64 = 10$$

$$10x^2 - 48x + 54 = 0$$

$$5x^2 - 24x + 27 = 0$$

$$x_{1,2} = \frac{12 \pm \sqrt{144-135}}{5} = \frac{12 \pm \sqrt{9}}{5} = \frac{12 \pm 3}{5} = \left\langle \begin{array}{l} \dfrac{9}{5} \\ \dfrac{15}{5} = 3 \end{array} \right.$$

$$\begin{cases} x = 3 \\ y = 3x - 8 = 9 - 8 = 1 \end{cases} \vee \begin{cases} x = \dfrac{9}{5} \\ y = 3x - 8 = 3 \cdot \dfrac{9}{5} - 8 = \dfrac{27}{5} - 8 = \dfrac{27-40}{5} = -\dfrac{13}{5} \end{cases}$$

4)

$$\begin{cases} 2x + 3y = 10 \\ xy + 4 = 0 \end{cases}$$

$$\begin{cases} x = \dfrac{10-3y}{2} \\ \dfrac{10-3y}{2} \cdot y + 4 = 0 \end{cases}$$

$$\frac{10y - 3y^2}{2} + 4 = 0$$

$$10y - 3y^2 + 8 = 0$$

$$3y^2 - 10y - 8 = 0$$

$$y_{1,2} = \frac{5 \pm \sqrt{25+24}}{3} = \frac{5 \pm \sqrt{49}}{3} = \frac{5 \pm 7}{3} = \left\langle \begin{array}{l} -\dfrac{2}{3} \\ \dfrac{12}{3} = 4 \end{array} \right.$$

$$\begin{cases} y = 4 \\ x = \dfrac{10-3y}{2} = \dfrac{10-12}{2} = \dfrac{-2}{2} = -1 \end{cases} \vee \begin{cases} y = -\dfrac{2}{3} \\ x = \dfrac{10-3y}{2} = \dfrac{10+2}{2} = \dfrac{12}{2} = 6 \end{cases}$$

$$\begin{cases} x = -1 \\ y = 4 \end{cases} \vee \begin{cases} x = 6 \\ y = -\dfrac{2}{3} \end{cases}$$

9)

$$\begin{cases} x^2+y^2-6x-2y=0 \\ x^2+y^2-8x-4y+16=0 \end{cases}$$

$$\begin{matrix} (1)-(2) \\ (1) \end{matrix} \begin{cases} 2x+2y-16=0 \\ x^2+y^2-6x-2y=0 \end{cases}$$

$$\begin{cases} x+y-8=0 \\ x^2+y^2-6x-2y=0 \end{cases}$$

$$\begin{cases} y=8-x \\ x^2+(8-x)^2-6x-2(8-x)=0 \end{cases}$$

$$\begin{cases} y=8-x \\ x^2+64-16x+x^2-6x-16+2x=0; \quad 2x^2-20x+48=0; \end{cases}$$

$$x^2-10x+24=0; \quad (x-4)(x-6)=0; \quad x=4 \ \vee \ x=6$$

$$\begin{cases} x=4 \\ y=8-4=4 \end{cases} \quad \begin{cases} x=6 \\ y=8-6=2 \end{cases}$$

12)

$$\begin{cases} x-y=1 \\ x^2+xy=a(2a-1) \end{cases}$$

$$\begin{cases} x=y+1 \\ (y+1)^2+(y+1)y=a(2a-1) \end{cases}$$

$$y^2+2y+1+y^2+y=2a^2-a$$

$$2y^2+3y+1-2a^2+a=0$$

$$y_{1,2}=\frac{-3\pm\sqrt{9-8\left(1-2a^2+a\right)}}{4}=\frac{-3\pm\sqrt{9-8+16a^2-8a}}{4}=$$

$$=\frac{-3\pm\sqrt{1+16a^2-8a}}{4}=\frac{-3\pm(1-4a)}{4}=\left[\begin{array}{l} \frac{-3-1+4a}{4}=\frac{4a-4}{4}=a-1 \\ \frac{-3+1-4a}{4}=\frac{-2-4a}{4}=-\frac{1+2a}{2} \end{array}\right.$$

$$\begin{cases} x=y+1=a\cancel{-1}\cancel{+1}=a \\ y=a-1 \end{cases} \qquad \begin{cases} x=y+1=-\frac{1+2a}{2}+1=\frac{-1-2a+2}{2}=\frac{1-2a}{2} \\ y=-\frac{1+2a}{2} \end{cases}$$

14)

$$\begin{cases} bx+ay=2ab \\ x^2+y^2=a^2+b^2 \end{cases} \qquad \begin{cases} x=\dfrac{2ab-ay}{b} \quad (b\neq 0) \\ \left(\dfrac{2ab-ay}{b}\right)^2+y^2=a^2+b^2 \end{cases}$$

$$\frac{4a^2b^2-4a^2by+a^2y^2}{b^2}+y^2=a^2+b^2$$

$$4a^2b^2-4a^2by+a^2y^2+b^2y^2=a^2b^2+b^4$$

$$a^2y^2+b^2y^2-4a^2by+3a^2b^2-b^4=0$$

$$\left(a^2+b^2\right)y^2-4a^2by+b^2\left(3a^2-b^2\right)=0$$

$$y_{1,2}=\frac{2a^2b\pm\sqrt{4a^4b^2-b^2\left(a^2+b^2\right)\left(3a^2-b^2\right)}}{a^2+b^2}=\frac{2a^2b\pm\sqrt{b^2\left[4a^4-\left(a^2+b^2\right)\left(3a^2-b^2\right)\right]}}{a^2+b^2}=$$

$$=\frac{2a^2b\pm\sqrt{b^2\left[4a^4-3a^4+a^2b^2-3a^2b^2+b^4\right]}}{a^2+b^2}=\frac{2a^2b\pm\sqrt{b^2\left[a^4-2a^2b^2+b^4\right]}}{a^2+b^2}=$$

$$=\frac{2a^2b\pm\sqrt{b^2\left(a^2-b^2\right)^2}}{a^2+b^2}=\frac{2a^2b\pm b\left(a^2-b^2\right)}{a^2+b^2}=\left\langle\begin{array}{l} \dfrac{2a^2b-a^2b+b^3}{a^2+b^2}=\dfrac{a^2b+b^3}{a^2+b^2}=\dfrac{b\cancel{\left(a^2+b^2\right)}}{\cancel{a^2+b^2}}=b \\ \dfrac{2a^2b+a^2b-b^3}{a^2+b^2}=\dfrac{3a^2b-b^3}{a^2+b^2}=\dfrac{b\left(3a^2-b^2\right)}{a^2+b^2} \end{array}\right.$$

$$\begin{cases} \boxed{x=}\dfrac{2ab-ay}{b}=\dfrac{2ab-ab}{b}=\dfrac{a\cancel{b}}{\cancel{b}}=\boxed{a} \\ \boxed{y=b} \end{cases}$$

oppure

$$\begin{cases} \boxed{x=}\dfrac{2ab-ay}{b}=\dfrac{2ab-a\dfrac{b\left(3a^2-b^2\right)}{a^2+b^2}}{b}=\dfrac{\dfrac{2ab\left(a^2+b^2\right)-ab\left(3a^2-b^2\right)}{a^2+b^2}}{b}= \\ \qquad =\dfrac{ab\left(2a^2+2b^2-3a^2+b^2\right)}{b\left(a^2+b^2\right)}=\dfrac{a\cancel{b}\left(-a^2+3b^2\right)}{\cancel{b}\left(a^2+b^2\right)}=\boxed{\dfrac{a\left(3b^2-a^2\right)}{a^2+b^2}} \\ \boxed{y=\dfrac{b\left(3a^2-b^2\right)}{a^2+b^2}} \end{cases}$$

Con $b=0$ *il sistema diventa* $\begin{cases} 0\cdot x+ay=2a\cdot 0 \\ x^2+y^2=a^2+0 \end{cases}$ $\begin{cases} ay=0 \\ x^2+y^2=a^2 \end{cases}$

e ora:

se $a\neq 0$, $\begin{cases} y=0 \\ x^2=a^2;\quad x=\pm a \end{cases}$ *che poi non è altro che la soluzione generale "particolarizzata"*

se $a=0$, $\begin{cases} 0\cdot y=0 \\ x^2+y^2=0 \end{cases}$ $\begin{cases} \text{la prima equazione è indeterminata...} \\ \text{... ma la seconda è verificata esclusivamente dalla coppia } x=0,\ y=0 \end{cases}$

e pertanto il sistema è verificato
esclusivamente dalla coppia $x=0$, $y=0$