CORREZIONE SISTEMI RISOLUBILI CON ARTIFICI

4)

$$\begin{cases} x^2+y^2+z^2=11 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases}$$

$$\begin{cases} (x+y+z)^2-2xy-2xz-2yz=11 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases}$$

$$\begin{cases} (x+y+z)^2-2(xy+xz+yz)=11 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases}$$

$$\begin{cases} (x+y+z)^2-2\cdot(-1)=11; \quad (x+y+z)^2=9; \quad x+y+z=\pm 3 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases}$$

$$\begin{cases} x+y+z=3 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases} \qquad \begin{cases} x+y+z=-3 \\ xy+xz+yz=-1 \\ x-y+z=5 \end{cases}$$

$$(1)-(3)\begin{cases} 2y=-2; \quad y=-1 \\ (1)\ x+y+z=3 \\ (2)\ xy+xz+yz=-1 \end{cases} \qquad (1)-(3)\begin{cases} 2y=-8; \quad y=-4 \\ (1)\ x+y+z=-3 \\ (2)\ xy+xz+yz=-1 \end{cases}$$

$$\begin{cases} y=-1 \\ x-1+z=3; \ x+z=4 \\ -x+xz-z=-1; \quad x+z-xz=1 \end{cases} \qquad \begin{cases} y=-4 \\ x-4+z=-3; \ x+z=1 \\ -4x+xz-4z=-1; \quad 4x+4z-xz=1; \ 4(x+z)-xz=1 \end{cases}$$

$$\begin{cases} y=-1 \\ x+z=4 \\ 4-xz=1; \ xz=3 \end{cases} \qquad \begin{cases} y=-4 \\ x+z=1 \\ 4-xz=1; \ xz=3 \end{cases}$$

$$\boxed{\begin{cases} y=-1 \\ x=1 \\ z=3 \end{cases}} \quad \boxed{\begin{cases} y=-1 \\ x=3 \\ z=1 \end{cases}} \qquad t^2-t+3=0 \quad \textit{impossibile in } \mathbb{R}$$

5)

$$\begin{cases} x-y+z=a \\ x-3y+2z+1=0 \\ x^2+z^2=2\left(y^2+1\right) \end{cases}$$

$$\begin{cases} x=a+y-z \\ a+y-z-3y+2z+1=0 \\ x^2+z^2=2y^2+2 \end{cases}$$

$$\begin{cases} x=a+y-z \\ -2y+z=-1-a; \quad z=2y-1-a \\ x^2+z^2=2y^2+2 \end{cases}$$

$$\begin{cases} z=2y-1-a \\ x=a+y-z=a+y-\left(2y-1-a\right)=a+y-2y+1+a=2a+1-y \\ x^2+z^2=2y^2+2 \end{cases}$$

$$\begin{cases} x=2a+1-y \\ z=2y-1-a \\ \left(2a+1-y\right)^2+\left(2y-1-a\right)^2=2y^2+2 \end{cases}$$

$$\begin{cases} x=2a+1-y \\ z=2y-1-a \\ 4a^2 \cancel{+1}+y^2+4a-4ay-2y+4y^2 \cancel{+1}+a^2-4y-4ay+2a=2y^2 \cancel{+2} \end{cases}$$

$$\begin{cases} x=2a+1-y \\ z=2y-1-a \\ 3y^2-8ay-6y+5a^2+6a=0; \;\; 3y^2-2\left(4a+3\right)y+a\left(5a+6\right)=0 \end{cases}$$

$$\begin{cases} x=2a+1-y \\ z=2y-1-a \\ y_{1,2}=\dfrac{4a+3\pm\sqrt{16a^2+24a+9-15a^2-18a}}{3}=\dfrac{4a+3\pm\sqrt{a^2+6a+9}}{3}= \\ \quad =\dfrac{4a+3\pm\left(a+3\right)}{3}=\left\langle \begin{array}{l} \dfrac{4a\cancel{+3}-a\cancel{-3}}{3}=\dfrac{3a}{3}=a \\ \dfrac{4a+3+a+3}{3}=\dfrac{5a+6}{3} \end{array} \right. \end{cases}$$

$$\begin{cases} \boxed{y=a} \\ \boxed{x=}2a+1-y=2a+1-a=\boxed{a+1} \\ \boxed{z=}2y-1-a=2a-1-a=\boxed{a-1} \end{cases} \qquad \begin{cases} \boxed{y=\dfrac{5a+6}{3}} \\ \boxed{x=}2a+1-y=2a+1-\dfrac{5a+6}{3}=\dfrac{6a+3-5a-6}{3}=\boxed{\dfrac{a-3}{3}} \\ \boxed{z=}2y-1-a=2\dfrac{5a+6}{3}-1-a=\dfrac{10a+12-3-3a}{3}=\boxed{\dfrac{7a+9}{3}} \end{cases}$$

6)

$$\begin{cases} x^2y+xy^2=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases}$$

$$\begin{cases} xy(x+y)=0;\ \ x=0 \vee y=0 \vee x+y=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases}$$

$$\begin{cases} x=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ x+y+z=1 \\ x^2+y^2+z^2=2 \end{cases}$$

$$\begin{cases} x=0 \\ y+z=1 \\ y^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x+z=1 \\ x^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ z=1 \\ x^2+y^2+1=2;\ \ x^2+y^2=1 \end{cases}$$

$$\begin{cases} x=0 \\ y=1-z \\ (1-z)^2+z^2=2 \end{cases} \vee \begin{cases} y=0 \\ x=1-z \\ (1-z)^2+z^2=2 \end{cases} \vee \begin{cases} x+y=0 \\ z=1 \\ (x+y)^2-2xy=1;\ \ -2xy=1;\ \ xy=-1/2 \end{cases}$$

$$\begin{cases} x=0 \\ y=1-z \\ z=\dfrac{1\pm\sqrt{3}}{2} \end{cases} \vee \begin{cases} y=0 \\ x=1-z \\ z=\dfrac{1\pm\sqrt{3}}{2} \end{cases} \vee \begin{cases} z=1 \\ y=-x \\ -x^2=-\dfrac{1}{2};\ \ x^2=\dfrac{1}{2};\ \ x=\pm\sqrt{\dfrac{1}{2}}=\pm\dfrac{1}{\sqrt{2}}=\pm\dfrac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} x=0 \\ y=\dfrac{1+\sqrt{3}}{2}, \\ z=\dfrac{1-\sqrt{3}}{2} \end{cases} \begin{cases} x=0 \\ y=\dfrac{1-\sqrt{3}}{2}, \\ z=\dfrac{1+\sqrt{3}}{2} \end{cases} \begin{cases} y=0 \\ x=\dfrac{1+\sqrt{3}}{2}, \\ z=\dfrac{1-\sqrt{3}}{2} \end{cases} \begin{cases} y=0 \\ x=\dfrac{1-\sqrt{3}}{2}, \\ z=\dfrac{1+\sqrt{3}}{2} \end{cases} \begin{cases} z=1 \\ y=\dfrac{\sqrt{2}}{2} \\ x=-\dfrac{\sqrt{2}}{2} \end{cases}, \quad \begin{cases} z=1 \\ y=-\dfrac{\sqrt{2}}{2} \\ x=\dfrac{\sqrt{2}}{2} \end{cases}$$

Ricapitolando, le soluzioni sono le 6 *terne*:

$$\left(0,\ \frac{1+\sqrt{3}}{2},\ \frac{1-\sqrt{3}}{2}\right) \qquad \left(0,\ \frac{1-\sqrt{3}}{2},\ \frac{1+\sqrt{3}}{2}\right)$$

$$\left(\frac{1+\sqrt{3}}{2},\ 0,\ \frac{1-\sqrt{3}}{2}\right) \qquad \left(\frac{1-\sqrt{3}}{2},\ 0,\ \frac{1+\sqrt{3}}{2}\right)$$

$$\left(-\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2},\ 1\right) \qquad \left(\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2},\ 1\right)$$

7)

$$\begin{cases} x+y-6z=0 \\ x^2+y^2+36z^2=56 \\ xyz=8 \end{cases}$$

$$\begin{cases} x+y=6z \\ xy=\dfrac{8}{z} \quad (z\neq 0) \\ (x+y)^2-2xy+36z^2=56 \end{cases}$$

$$\begin{cases} x+y=6z \\ xy=\dfrac{8}{z} \\ 36z^2-\dfrac{16}{z}+36z^2=56 \end{cases}$$

$72z^2-\dfrac{16}{z}=56;\quad 72z^3-16=56z;\quad 9z^3-2=7z;$

$9z^3-7z-2=0$

$P(1)=0$

	9	0	−7	−2
1		9	9	2
	9	9	2	0

$(z-1)(9z^2+9z+2)=0$

$$z=1;\quad z=\frac{-9\pm\sqrt{81-72}}{18}=\frac{-9\pm\sqrt{9}}{18}=\frac{-9\pm 3}{18}=\left\langle \begin{matrix} -\dfrac{12}{18}=-\dfrac{2}{3} \\ -\dfrac{6}{18}=-\dfrac{1}{3} \end{matrix} \right.$$

$$\begin{cases} z=1 \\ x+y=6z=6\cdot 1=6 \\ xy=\dfrac{8}{z}=\dfrac{8}{1}=8 \end{cases} \qquad \begin{cases} z=-\dfrac{2}{3} \\ x+y=6z=6\cdot\left(-\dfrac{2}{3}\right)=-4 \\ xy=\dfrac{8}{z}=\dfrac{8}{-\frac{2}{3}}=-12 \end{cases} \qquad \begin{cases} z=-\dfrac{1}{3} \\ x+y=6z=6\cdot\left(-\dfrac{1}{3}\right)=-2 \\ xy=\dfrac{8}{z}=\dfrac{8}{-\frac{1}{3}}=-24 \end{cases}$$

$$\boxed{\begin{cases} z=1 \\ x=2 \\ y=4 \end{cases}} \quad \boxed{\begin{cases} z=1 \\ x=4 \\ y=2 \end{cases}} \qquad \boxed{\begin{cases} z=-\dfrac{2}{3} \\ x=-6 \\ y=2 \end{cases}} \quad \boxed{\begin{cases} z=-\dfrac{2}{3} \\ x=2 \\ y=-6 \end{cases}} \qquad \boxed{\begin{cases} z=-\dfrac{1}{3} \\ x=-6 \\ y=4 \end{cases}} \quad \boxed{\begin{cases} z=-\dfrac{1}{3} \\ x=4 \\ y=-6 \end{cases}}$$