n) ESPRESSIONI VARIE - SVOLGIMENTI

$$\boxed{15)\ \sqrt[3]{\frac{t^2-4}{t-1}}\cdot\sqrt{\frac{t^2-3t+2}{t+2}}} = \sqrt[3]{\frac{(t+2)(t-2)}{t-1}}\cdot\sqrt{\frac{(t-1)(t-2)}{t+2}} =$$

$$= \sqrt[6]{\frac{(t+2)^2(t-2)^2}{(t-1)^2}}\cdot\sqrt[6]{\frac{(t-1)^3(t-2)^3}{(t+2)^3}} = \sqrt[6]{\frac{\cancel{(t+2)^2}\,(t-2)^2}{\cancel{(t-1)^2}}\cdot\frac{(t-1)^{\cancel{3}}\,(t-2)^3}{(t+2)^{\cancel{3}}}} = \boxed{\sqrt[6]{\frac{(t-1)(t-2)^5}{t+2}}}$$

$$\boxed{16)\ \frac{\sqrt{2}\cdot\sqrt[3]{3}}{\sqrt[4]{4}\cdot\sqrt[5]{5}}} = \frac{\sqrt{2}\cdot\sqrt[3]{3}}{\sqrt[\cancel{4}^{2}]{2^{\cancel{2}}}\cdot\sqrt[5]{5}} = \frac{\cancel{\sqrt{2}}\cdot\sqrt[3]{3}}{\cancel{\sqrt{2}}\cdot\sqrt[5]{5}} = \frac{\sqrt[15]{3^5}}{\sqrt[15]{5^3}} = \sqrt[15]{\frac{243}{125}} = \boxed{\sqrt[15]{1{,}944}\approx 1{,}045}$$

$$\boxed{17)\ \sqrt{x\sqrt[3]{x}}:\sqrt[3]{x\sqrt{x}}} = \sqrt{\sqrt[3]{x^4}}:\sqrt[3]{\sqrt{x^3}} = \sqrt[6]{x^4}:\sqrt[6]{x^3} = \boxed{\sqrt[6]{x}}$$

$$\boxed{18)\ \frac{\left(\sqrt[3]{(x-y)\sqrt[4]{(x-y)}}\right)^8}{x^3-y^3}} = \frac{\left(\sqrt[3]{\sqrt[4]{(x-y)^4(x-y)}}\right)^8}{x^3-y^3} = \frac{\left(\sqrt[\cancel{12}^{3}]{(x-y)^5}\right)^{\cancel{8}^{2}}}{x^3-y^3} =$$

$$= \frac{\sqrt[3]{(x-y)^{10}}}{x^3-y^3} = \frac{(x-y)^{\cancel{3}^{2}}\sqrt[3]{x-y}}{\cancel{(x-y)}\left(x^2+xy+y^2\right)} = \boxed{\frac{(x-y)^2\sqrt[3]{x-y}}{x^2+xy+y^2}}$$

$$\boxed{19)\ \sqrt[k]{a^p}\cdot\sqrt[p]{a^k}} = \sqrt[kp]{a^{p^2}}\cdot\sqrt[kp]{a^{k^2}} = \sqrt[kp]{a^{p^2}\cdot a^{k^2}} = \boxed{\sqrt[kp]{a^{p^2+k^2}}}$$

$$\boxed{20)\ \frac{a}{b}\sqrt{\frac{b}{a}}\cdot\sqrt[4]{\frac{b}{a}}\cdot\sqrt[8]{ab}-\sqrt[8]{\frac{a^3}{b}}} = \frac{a}{b}\sqrt[8]{\frac{b^4}{a^4}\cdot\frac{b^2}{a^2}\cdot ab}-\sqrt[8]{\frac{a^3}{b}} = \frac{a}{b}\sqrt[8]{\frac{b^7}{a^5}}-\sqrt[8]{\frac{a^3}{b}} =$$

$$= \sqrt[8]{\frac{a^8}{b^8}\cdot\frac{b^7}{a^5}}-\sqrt[8]{\frac{a^3}{b}} = \sqrt[8]{\frac{a^3}{b}}-\sqrt[8]{\frac{a^3}{b}} = \boxed{0}$$

$$\boxed{21)\ \sqrt{160}+\sqrt{250}+\sqrt{360}-\sqrt{2250}} = \sqrt{16\cdot10}+\sqrt{25\cdot10}+\sqrt{36\cdot10}-\sqrt{225\cdot10} =$$

$$= 4\sqrt{10}+5\sqrt{10}+6\sqrt{10}-15\sqrt{10} = \boxed{0}$$

$$\boxed{22)\ \frac{\sqrt{108}+\sqrt{243}}{\sqrt[3]{81}-2\sqrt[3]{3}}} = \frac{\sqrt{2^2\cdot3^3}+\sqrt{3^5}}{\sqrt[3]{3^4}-2\sqrt[3]{3}} = \frac{2\cdot3\sqrt{3}+3^2\sqrt{3}}{3\sqrt[3]{3}-2\sqrt[3]{3}} = \frac{15\sqrt{3}}{\sqrt[3]{3}} = 15\sqrt[6]{\frac{3^3}{3^2}} = \boxed{15\sqrt[6]{3}}$$

$$\boxed{23)\ \sqrt{x^3-3x^2+3x-1}+\sqrt{9x-9}} = \sqrt{(x-1)^3}+\sqrt{9(x-1)} = (x-1)\sqrt{x-1}+3\sqrt{x-1} = \boxed{(x+2)\sqrt{x-1}}$$

$$\boxed{24)\ \frac{\sqrt[4]{243}+\sqrt[4]{48}}{\sqrt{13^2-12^2}}} = \frac{\sqrt[4]{3^5}+\sqrt[4]{2^4\cdot3}}{\sqrt{169-144}} = \frac{3\sqrt[4]{3}+2\sqrt[4]{3}}{\sqrt{25}} = \frac{3\sqrt[4]{3}+2\sqrt[4]{3}}{5} = \frac{5\sqrt[4]{3}}{5} = \boxed{\sqrt[4]{3}}$$

Occhio! GRAVE ERRORE
SAREBBE STATO scrivere
$\sqrt{13^2-12^2}=\sqrt{13-12}=1$!!!!

25) $\sqrt{\left(2+\sqrt{3}\right)^2+\left(2\sqrt{3}-1\right)^2} = \sqrt{4+3+\cancel{4\sqrt{3}}+12+1\cancel{-4\sqrt{3}}} = \sqrt{20} = \boxed{2\sqrt{5}}$

Occhio!!! *SAREBBE STATO GRAVE ERRORE mandar via la radice coi due quadrati*!

26) $\left(\sqrt[4]{1+m}-\sqrt[4]{1-m}\right)\left(\sqrt[4]{1+m}+\sqrt[4]{1-m}\right)\left(\sqrt{1+m}+\sqrt{1-m}\right) =$

$$= \left[\left(\sqrt[\cancel{4}\,2]{1+m}\right)^{\cancel{2}} - \left(\sqrt[\cancel{4}\,2]{1-m}\right)^{\cancel{2}}\right]\left(\sqrt{1+m}+\sqrt{1-m}\right) =$$

$$= \left(\sqrt{1+m}-\sqrt{1-m}\right)\left(\sqrt{1+m}+\sqrt{1-m}\right) = \left(\sqrt{1+m}\right)^2-\left(\sqrt{1-m}\right)^2 = (1+m)-(1-m) = 1+m-1+m = \boxed{2m}$$

27) $\left(2\sqrt{2}-\sqrt{3}-1\right)^2-\left(2\sqrt{3}-3\sqrt{2}\right)^2+2\sqrt{2}\left(2+\frac{9}{2}\sqrt{2}-4\sqrt{3}-\sqrt{\frac{3}{2}}\right) =$

$$= 8+3+1-4\sqrt{6}\cancel{-4\sqrt{2}}\cancel{+2\sqrt{3}}-\left(12-12\sqrt{6}+18\right)\cancel{+4\sqrt{2}}+18-8\sqrt{6}\cancel{-2\sqrt{3}} =$$

$$= \cancel{30}\cancel{-12\sqrt{6}}\cancel{-12}\cancel{+12\sqrt{6}}\cancel{-18} = \boxed{0}$$

28) $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{\left(\sqrt{a}+\sqrt{b}\right)^2-\left(\sqrt{a}-\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)} = \frac{a+b+2\sqrt{ab}-\left(a+b-2\sqrt{ab}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)} =$

$$= \frac{a+b+2\sqrt{ab}-a-b+2\sqrt{ab}}{\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)} = \boxed{\frac{4\sqrt{ab}}{a-b}}$$

(In alternativa, si poteva procedere razionalizzando le singole frazioni...)

29) $\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}+\sqrt{\frac{x^2}{y^2}-1} = \frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}\cdot\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\sqrt{\frac{x^2-y^2}{y^2}} =$

$$= \frac{\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}{x+y-(x-y)}+\frac{1}{y}\sqrt{x^2-y^2} = \frac{x\cancel{+y}+x\cancel{-y}-2\sqrt{x^2-y^2}}{2y}+\frac{1}{y}\sqrt{x^2-y^2} =$$

$$= \frac{2x-2\sqrt{x^2-y^2}}{2y}+\frac{1}{y}\sqrt{x^2-y^2} = \frac{\cancel{2}\left(x-\sqrt{x^2-y^2}\right)}{\cancel{2}y}+\frac{\sqrt{x^2-y^2}}{y} = \frac{x\cancel{-\sqrt{x^2-y^2}}\cancel{+\sqrt{x^2-y^2}}}{y} = \boxed{\frac{x}{y}}$$

30) $\frac{2\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}-\frac{x}{1-\sqrt{1-x^2}} = \frac{2\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}-\frac{x}{1-\sqrt{1-x^2}}\cdot\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} =$

$$= \frac{2\sqrt{1+x}\left(\sqrt{1+x}+\sqrt{1-x}\right)}{1+x-(1-x)}-\frac{x\left(1+\sqrt{1-x^2}\right)}{1-\left(1-x^2\right)} = \frac{\cancel{2}\sqrt{1+x}\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\cancel{2}x}-\frac{\cancel{x}\left(1+\sqrt{1-x^2}\right)}{x^{\cancel{2}}} =$$

$$= \frac{\cancel{1}+x\cancel{+\sqrt{1-x^2}}\cancel{-1}\cancel{-\sqrt{1-x^2}}}{x} = \frac{x}{x} = \boxed{1}$$

31) $\frac{\frac{1}{1+\sqrt{x}}-1}{\left(1+\sqrt[4]{x}\right)\left(1-\sqrt[4]{x}\right)} = \frac{\frac{1-1-\sqrt{x}}{1+\sqrt{x}}}{1-\left(\sqrt[\cancel{4}\,2]{x}\right)^{\cancel{2}}} = -\frac{\sqrt{x}}{1+\sqrt{x}}\cdot\frac{1}{1-\sqrt{x}} = -\frac{\sqrt{x}}{1-x} = \boxed{\frac{\sqrt{x}}{x-1}}$

$$\boxed{32)\quad \frac{x^2-x}{\sqrt{1-x^2}+x-1}} = \frac{x(x-1)}{\sqrt{1-x^2}+x-1}\cdot\frac{\sqrt{1-x^2}-(x-1)}{\sqrt{1-x^2}-(x-1)} = \frac{x(x-1)\left[\sqrt{1-x^2}-x+1\right]}{1-x^2-(x-1)^2} =$$

$$= \frac{x(x-1)\left[\sqrt{1-x^2}-x+1\right]}{\cancel{1}-x^2-x^2+2x\cancel{-1}} = \frac{x(x-1)\left[\sqrt{1-x^2}-x+1\right]}{2x-2x^2} = \frac{\cancel{x}\,\overset{-1}{\cancel{(x-1)}}\left[\sqrt{1-x^2}-x+1\right]}{2\cancel{x}\,\cancel{(1-x)}} =$$

$$= -\frac{\sqrt{1-x^2}-x+1}{2} = \boxed{\frac{x-1-\sqrt{1-x^2}}{2}}$$

$$\boxed{33)\quad \frac{\sqrt2}{2\sqrt2+3}+\frac{\sqrt3}{2-\sqrt3}+1} = \frac{\sqrt2}{2\sqrt2+3}\cdot\frac{2\sqrt2-3}{2\sqrt2-3}+\frac{\sqrt3}{2-\sqrt3}\cdot\frac{2+\sqrt3}{2+\sqrt3}+1 = \frac{\sqrt2\left(2\sqrt2-3\right)}{8-9}+\frac{\sqrt3\left(2+\sqrt3\right)}{4-3}+1 =$$

$$= -\sqrt2\left(2\sqrt2-3\right)+\sqrt3\left(2+\sqrt3\right)+1 = \cancel{-4}+3\sqrt2+2\sqrt3\cancel{+3}\cancel{+1} = \boxed{3\sqrt2+2\sqrt3}$$

$$\boxed{34)\quad \sqrt[3]{2\sqrt7-1}\cdot\sqrt[3]{2\sqrt7+1}-\sqrt{16-6\sqrt7}-\left(\sqrt{\sqrt7}\right)^2} = \sqrt[3]{\left(2\sqrt7-1\right)\left(2\sqrt7+1\right)}-\sqrt{\left(3-\sqrt7\right)^2}-\sqrt7 =$$

$$= \sqrt[3]{\left(2\sqrt7\right)^2-1^2}-\left(3-\sqrt7\right)-\sqrt7 = \sqrt[3]{28-1}-3\cancel{+\sqrt7}\cancel{-\sqrt7} = \sqrt[3]{27}-3 = 3-3 = \boxed{0}$$

$$\boxed{35)\quad \frac{\sqrt{4+2\sqrt3}+\sqrt{4-2\sqrt3}}{\sqrt{2\sqrt3+3}\cdot\sqrt{2\sqrt3-3}}} = \frac{\sqrt{\left(\sqrt3+1\right)^2}+\sqrt{\left(\sqrt3-1\right)^2}}{\sqrt{\left(2\sqrt3+3\right)\left(2\sqrt3-3\right)}} = \frac{\sqrt3+1+\sqrt3-1}{\sqrt{12-9}} = \frac{2\sqrt3}{\sqrt3} = \boxed{2}$$

Scomposizione con Ruffini

$$\boxed{36)\quad \frac{\sqrt{x^3-3x-2}-\sqrt{x^3-6x^2+12x-8}}{3}} = \frac{\sqrt{(x+1)^2(x-2)}-\sqrt{(x-2)^3}}{3} =$$

$$= \frac{(x+1)\sqrt{x-2}-(x-2)\sqrt{x-2}}{3} = \frac{(x+1-x+2)\sqrt{x-2}}{3} = \frac{3\sqrt{x-2}}{3} = \boxed{\sqrt{x-2}}$$

$$\boxed{37)\quad \left(\sqrt[4]{a}+\frac{\sqrt[3]{a^2}}{\sqrt{a\sqrt a}}\right)\left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}}\right)+\sqrt[3]{a}} = \left(\sqrt[4]{a}+\frac{\sqrt[3]{a^2}}{\sqrt{\sqrt{a^3}}}\right)\left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}}\right)+\sqrt[3]{a} =$$

$$= \left(\sqrt[4]{a}+\frac{\sqrt[3]{a^2}}{\sqrt[4]{a^3}}\right)\left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}}\right)+\sqrt[3]{a} = \left(\frac{\sqrt[4]{a^4}+\sqrt[3]{a^2}}{\sqrt[4]{a^3}}\right)\left(\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}}\right)+\sqrt[3]{a} = \frac{a+\sqrt[3]{a^2}}{\sqrt[4]{a^3}}\cdot\frac{a-\sqrt[3]{a^2}}{\sqrt[4]{a}}+\sqrt[3]{a} =$$

$$= \frac{a^2-\sqrt[3]{a^4}}{\sqrt[4]{a^4}}+\sqrt[3]{a} = \frac{a^{\cancel{2}}-\cancel{a}\sqrt[3]{a}}{\cancel{a}}+\sqrt[3]{a} = a-\sqrt[3]{a}+\sqrt[3]{a} = \boxed{a}$$

$$\boxed{38)\quad \sqrt{\frac34}+\frac34\sqrt{\frac43}} = \sqrt{\frac34}+\sqrt{\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{16}}}\cdot\frac{\cancel{4}}{\cancel{3}}} = \sqrt{\frac34}+\sqrt{\frac34} = 2\sqrt{\frac34} = \sqrt{\cancel{4}\cdot\frac{3}{\cancel{4}}} = \boxed{\sqrt3}$$

oppure:

$$\sqrt{\frac34}+\frac34\sqrt{\frac43} = \frac{\sqrt3}{\sqrt4}+\frac34\cdot\frac{\sqrt4}{\sqrt3} = \frac{\sqrt3}{2}+\frac{3}{\underset{2}{\cancel{4}}}\cdot\frac{\cancel{2}}{\sqrt3} = \frac{\sqrt3}{2}+\frac{\sqrt3\cdot\cancel{\sqrt3}}{2\cancel{\sqrt3}} = \frac{\cancel{2}\sqrt3}{\cancel{2}} = \boxed{\sqrt3}$$

$$39)\ \frac{a\sqrt[3]{a}+\sqrt[3]{a}}{a\sqrt[3]{b}+\sqrt[3]{b}}-\sqrt[3]{\frac{a}{b}}=\frac{\cancel{(a+1)}\sqrt[3]{a}}{\cancel{(a+1)}\sqrt[3]{b}}-\sqrt[3]{\frac{a}{b}}=\frac{\sqrt[3]{a}}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}=\boxed{0}$$

$$40)\ \frac{1}{4}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}-\frac{1}{4}=\frac{1}{4}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}-\frac{1}{4}=$$

$$=\frac{1}{4}\cdot\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{3-2}+\frac{\sqrt{6}}{2}-\frac{1}{4}=$$

$$=\frac{1}{4}\cdot\left(3+2-2\sqrt{6}\right)+\frac{\sqrt{6}}{2}-\frac{1}{4}=\frac{5}{4}\cancel{-\frac{\sqrt{6}}{2}}\cancel{+\frac{\sqrt{6}}{2}}-\frac{1}{4}=\frac{4}{4}=\boxed{1}$$

$$41)\ \frac{\left(\sqrt{3}+1\right)^2-\left(\sqrt{2}-1\right)^2-1}{\sqrt{12}+\sqrt{8}}=\frac{\cancel{3}\cancel{+1}+2\sqrt{3}\cancel{-2}\cancel{-1}+2\sqrt{2}\cancel{-1}}{2\sqrt{3}+2\sqrt{2}}=\frac{\cancel{2\sqrt{3}+2\sqrt{2}}}{\cancel{2\sqrt{3}+2\sqrt{2}}}=\boxed{1}$$

$$42)\ \frac{\sqrt{\sqrt{9+4\sqrt{2}}+\sqrt{9-4\sqrt{2}}}}{2}=\frac{\sqrt{\sqrt{\left(2\sqrt{2}+1\right)^2}+\sqrt{\left(2\sqrt{2}-1\right)^2}}}{2}=$$

$$=\frac{\sqrt{2\sqrt{2}\cancel{+1}+2\sqrt{2}\cancel{-1}}}{2}=\frac{\sqrt{4\sqrt{2}}}{2}=\frac{\cancel{2}\sqrt{\sqrt{2}}}{\cancel{2}}=\boxed{\sqrt[4]{2}}$$

$$43)\ \left(\frac{\sqrt[4]{x}}{2}-\frac{2}{\sqrt[4]{x}}\right):\sqrt[4]{x}+2\frac{\sqrt{x}}{x}=\frac{\sqrt{x}-4}{2\sqrt[4]{x}}:\sqrt[4]{x}+\frac{2\sqrt{x}}{x}=\frac{\sqrt{x}-4}{2\sqrt[4]{x}}\cdot\frac{1}{\sqrt[4]{x}}+\frac{2\sqrt{x}}{x}=$$

$$=\frac{\sqrt{x}-4}{2\sqrt{x}}+\frac{2\sqrt{x}}{x}=\frac{\sqrt{x}\left(\sqrt{x}-4\right)+4\sqrt{x}}{2x}=$$

$$=\frac{x\cancel{-4\sqrt{x}}\cancel{+4\sqrt{x}}}{2x}=\frac{x}{2x}=\boxed{\frac{1}{2}}$$

$$44)\ \left(\frac{1}{\sqrt{1-w}}+\frac{1}{\sqrt{1+w}}\right)^2-\frac{2}{\sqrt{1-w^2}}=\left(\frac{\sqrt{1+w}+\sqrt{1-w}}{\sqrt{1-w}\cdot\sqrt{1+w}}\right)^2-\frac{2}{\sqrt{1-w^2}}\cdot\frac{\sqrt{1-w^2}}{\sqrt{1-w^2}}=$$

$$=\frac{1\cancel{+w}+1\cancel{-w}+2\sqrt{1-w^2}}{(1-w)(1+w)}-\frac{2\sqrt{1-w^2}}{1-w^2}=$$

$$=\frac{2+2\sqrt{1-w^2}}{1-w^2}-\frac{2\sqrt{1-w^2}}{1-w^2}=$$

$$=\frac{2\cancel{+2\sqrt{1-w^2}}\cancel{-2\sqrt{1-w^2}}}{1-w^2}=\boxed{\frac{2}{1-w^2}}$$

$$45)\ \frac{\sqrt{1+x}}{\sqrt{x}}-\frac{2\sqrt{x}}{\sqrt{1+x}}-\left(\frac{1}{\sqrt{x}}-\sqrt{x}\right):\sqrt{1+x}=\frac{1+x-2x}{\sqrt{x}\cdot\sqrt{1+x}}-\frac{1-x}{\sqrt{x}}\cdot\frac{1}{\sqrt{1+x}}=\frac{1-x}{\sqrt{x}\cdot\sqrt{1+x}}-\frac{1-x}{\sqrt{x}\sqrt{1+x}}=\boxed{0}$$

$$46)\ \frac{\sqrt[3]{\frac{a}{b}}\cdot\sqrt[4]{\frac{b}{a}}}{\sqrt{\sqrt{\sqrt[3]{\frac{a}{b}}}}}=\frac{\sqrt[12]{\frac{a^4}{b^4}}\cdot\sqrt[12]{\frac{b^3}{a^3}}}{\sqrt[12]{\frac{a}{b}}}=\frac{\sqrt[12]{\frac{a^{\cancel{4}}}{b^{\cancel{4}}}\cdot\frac{\cancel{b^3}}{\cancel{a^3}}}}{\sqrt[12]{\frac{a}{b}}}=\frac{\sqrt[12]{\frac{a}{b}}}{\sqrt[12]{\frac{a}{b}}}=\boxed{1}$$

$$\boxed{47)\ \sqrt{\frac{\left(\sqrt{a}\right)^4-\left(\sqrt{b}\right)^4}{b\left(\frac{\sqrt[3]{a}}{\sqrt{b}}\right)^6-1}}} = \sqrt{\frac{a^2-b^2}{\cancel{b}\cdot\frac{a^2}{b^{\cancel{3}2}}-1}} = \sqrt{\frac{a^2-b^2}{\frac{a^2-b^2}{b^2}}} = \sqrt{\cancel{\left(a^2-b^2\right)}\cdot\frac{b^2}{\cancel{a^2-b^2}}} = \sqrt{b^2} = \boxed{b}$$

$$\boxed{48)\ \frac{1}{x+\sqrt[3]{1-x^3}}} = \frac{1}{x+\sqrt[3]{1-x^3}}\cdot\frac{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{\left(1-x^3\right)^2}}{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{\left(1-x^3\right)^2}} =$$

$$= \frac{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{\left(1-x^3\right)^2}}{x^3+\left(\sqrt[3]{1-x^3}\right)^3} =$$

$$= \frac{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{\left(1-x^3\right)^2}}{\cancel{x^3}+1-\cancel{x^3}}$$

$$\boxed{x^2-x\sqrt[3]{1-x^3}+\sqrt[3]{\left(1-x^3\right)^2}}$$

$$\boxed{49)\ \frac{2}{\sqrt[4]{a+1}+\sqrt[4]{a-1}}} = \frac{2}{\sqrt[4]{a+1}+\sqrt[4]{a-1}}\cdot\frac{\sqrt[4]{a+1}-\sqrt[4]{a-1}}{\sqrt[4]{a+1}-\sqrt[4]{a-1}} = \frac{2\left(\sqrt[4]{a+1}-\sqrt[4]{a-1}\right)}{\left(\sqrt[\cancel{4}2]{a+1}\right)^{\cancel{2}}-\left(\sqrt[\cancel{4}2]{a-1}\right)^{\cancel{2}}} =$$

$$= \frac{2\left(\sqrt[4]{a+1}-\sqrt[4]{a-1}\right)}{\sqrt{a+1}-\sqrt{a-1}}\cdot\frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}} = \frac{2\left(\sqrt[4]{a+1}-\sqrt[4]{a-1}\right)\left(\sqrt{a+1}+\sqrt{a-1}\right)}{\cancel{a}+1-\cancel{a}+1} =$$

$$= \frac{\cancel{2}\left(\sqrt[4]{a+1}-\sqrt[4]{a-1}\right)\left(\sqrt{a+1}+\sqrt{a-1}\right)}{\cancel{2}} = \boxed{\left(\sqrt[4]{a+1}-\sqrt[4]{a-1}\right)\left(\sqrt{a+1}+\sqrt{a-1}\right)}$$

$$\boxed{50)\ \frac{\sqrt{ac}-\sqrt{c}}{a-\sqrt{a}+\sqrt{ab}-\sqrt{b}}} = \frac{\sqrt{c}\left(\sqrt{a}-1\right)}{\sqrt{a}\left(\sqrt{a}-1\right)+\sqrt{b}\left(\sqrt{a}-1\right)} = \frac{\sqrt{c}\,\cancel{\left(\sqrt{a}-1\right)}}{\cancel{\left(\sqrt{a}-1\right)}\left(\sqrt{a}+\sqrt{b}\right)} = \boxed{\frac{\sqrt{c}}{\sqrt{a}+\sqrt{b}}}$$

$$\boxed{51)\ \frac{\sqrt{10}+\sqrt{6}-\sqrt{5}-\sqrt{3}}{2\sqrt{6}-2\sqrt{3}-\sqrt{2}+1}} = \frac{\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)-\left(\sqrt{5}+\sqrt{3}\right)}{2\sqrt{3}\left(\sqrt{2}-1\right)-\left(\sqrt{2}-1\right)} = \frac{\left(\sqrt{5}+\sqrt{3}\right)\cancel{\left(\sqrt{2}-1\right)}}{\cancel{\left(\sqrt{2}-1\right)}\left(2\sqrt{3}-1\right)} = \boxed{\frac{\sqrt{5}+\sqrt{3}}{2\sqrt{3}-1}}$$

$$\boxed{52)\ \frac{\sqrt{3}+\sqrt{2}+1}{\sqrt{3}-\sqrt{2}+1}} = \frac{\sqrt{3}+\sqrt{2}+1}{\sqrt{3}-\left(\sqrt{2}-1\right)}\cdot\frac{\sqrt{3}+\left(\sqrt{2}-1\right)}{\sqrt{3}+\left(\sqrt{2}-1\right)} = \frac{\left(\sqrt{3}+\sqrt{2}+1\right)\left(\sqrt{3}+\sqrt{2}-1\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}-1\right)^2} =$$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)^2-1^2}{3-\left(2+1-2\sqrt{2}\right)} = \frac{3+2+2\sqrt{6}-1}{2\sqrt{2}} = \frac{4+2\sqrt{6}}{2\sqrt{2}} = \frac{\cancel{2}\left(2+\sqrt{6}\right)}{\cancel{2}\sqrt{2}} = \frac{2+\sqrt{6}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{2\sqrt{2}+\sqrt{12}}{2} = \frac{2\sqrt{2}+2\sqrt{3}}{2} = \boxed{\sqrt{2}+\sqrt{3}\approx 1{,}414+1{,}732=3{,}146}$$