ESERCIZI CON ESPONENTI FRAZIONARI - SVOLGIMENTI

1) $8^{\frac{2}{3}} + 9^{\frac{1}{2}} = \sqrt[3]{8^2} + \sqrt{9} = \sqrt[3]{64} + 3 = 4 + 3 = 7$

2) $4^{\frac{3}{2}} - 8^{\frac{1}{3}} - 3 \cdot 16^{\frac{1}{4}} = \sqrt{4^3} - \sqrt[3]{8} - 3\sqrt[4]{16} = \sqrt{64} - 2 - 3 \cdot 2 = 8 - 2 - 6 = 0$

3) $\left[30\left(25^{-\frac{1}{2}} + 9^{-\frac{1}{2}} \right) \right]^{\frac{1}{4}} = \left[30\left(\frac{1}{\sqrt{25}} + \frac{1}{\sqrt{9}} \right) \right]^{\frac{1}{4}} = \sqrt[4]{30\left(\frac{1}{5} + \frac{1}{3} \right)} = \sqrt[4]{\overset{2}{\cancel{30}} \cdot \frac{8}{\cancel{15}}} = \sqrt[4]{16} = 2$

4) $\left(\frac{8}{27} \right)^{-\frac{1}{3}} - \left(\frac{1}{4} \right)^{\frac{1}{2}} = \left(\frac{27}{8} \right)^{\frac{1}{3}} - \sqrt{\frac{1}{4}} = \sqrt[3]{\frac{27}{8}} - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$

5) $64^{\frac{5}{6}} - 81^{\frac{3}{4}} = \sqrt[6]{64^5} - \sqrt[4]{81^3} = \left(\sqrt[6]{64} \right)^5 - \left(\sqrt[4]{81} \right)^3 = 2^5 - 3^3 = 32 - 27 = 5$

6) $2^{\frac{1}{2}} + 2^{-\frac{1}{2}} = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

7) $3^{\frac{1}{4}} \cdot 27^{\frac{1}{4}} = (3 \cdot 27)^{\frac{1}{4}} = 81^{\frac{1}{4}} = \sqrt[4]{81} = 3$ 8) $5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{6}} = 5^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = 5^{\frac{3+2+1}{6}} = 5^{\frac{6}{6}} = 5$

9) $\left(81^{\frac{1}{2}} \right)^{-\frac{1}{2}} = 81^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$ 10) $2^{-\frac{2}{3}} \cdot 4^{-\frac{2}{3}} = (2 \cdot 4)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$

11) $\sqrt{a} \cdot \sqrt[3]{a^2} \cdot \sqrt[4]{a^3} = a^{\frac{1}{2}} \cdot a^{\frac{2}{3}} \cdot a^{\frac{3}{4}} = a^{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}} = a^{\frac{23}{12}}$ 12) $\frac{\sqrt[4]{b^5}}{b \cdot \sqrt[8]{b}} = \frac{b^{\frac{5}{4}}}{b \cdot b^{\frac{1}{8}}} = \frac{b^{\frac{5}{4}}}{b^{1+\frac{1}{8}}} = \frac{b^{\frac{5}{4}}}{b^{\frac{9}{8}}} = b^{\frac{5}{4}-\frac{9}{8}} = b^{\frac{1}{8}}$

13) $\left(\frac{\sqrt{\sqrt{x^3}} \cdot \sqrt[3]{x}}{x} \right)^{12} = \left[\frac{\left(x^{\frac{3}{2}} \right)^{\frac{1}{2}} \cdot x^{\frac{1}{3}}}{x} \right]^{12} = \left(\frac{x^{\frac{3}{4}} \cdot x^{\frac{1}{3}}}{x} \right)^{12} = \left(x^{\frac{3}{4}+\frac{1}{3}-1} \right)^{12} = \left(x^{\frac{1}{12}} \right)^{12} = x^1 = x$

14) $\sqrt[3]{\frac{\sqrt[10]{a} \cdot \sqrt[5]{a^2}}{\sqrt{a}}} = \left(\frac{a^{\frac{1}{10}} \cdot a^{\frac{2}{5}}}{a^{\frac{1}{2}}} \right)^{\frac{1}{3}} = \left(a^{\frac{1}{10}+\frac{2}{5}-\frac{1}{2}} \right)^{\frac{1}{3}} = \left(a^{\frac{1+4-5}{10}} \right)^{\frac{1}{3}} = \left(a^0 \right)^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1$

15) $\left(\sqrt[4]{a} + \sqrt[4]{b} \right)\left(\sqrt[4]{a} - \sqrt[4]{b} \right) = \left(a^{\frac{1}{4}} + b^{\frac{1}{4}} \right)\left(a^{\frac{1}{4}} - b^{\frac{1}{4}} \right) = \left(a^{\frac{1}{4}} \right)^2 - \left(b^{\frac{1}{4}} \right)^2 = a^{\frac{1}{4} \cdot 2} - b^{\frac{1}{4} \cdot 2} = a^{\frac{1}{2}} - b^{\frac{1}{2}} = \sqrt{a} - \sqrt{b}$

16) $\frac{y^2 \cdot \sqrt{3x} \cdot \sqrt[4]{9xy}}{\sqrt[3]{y^2}} = \frac{y^2 \cdot (3x)^{\frac{1}{2}} \cdot (9xy)^{\frac{1}{4}}}{y^{\frac{2}{3}}} = y^2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot 9^{\frac{1}{4}} \cdot x^{\frac{1}{4}} \cdot y^{\frac{1}{4}} \cdot y^{-\frac{2}{3}} = $

         $= 3^{\frac{1}{2}} \cdot \left(3^{\cancel{2}} \right)^{\frac{1}{\cancel{4}_2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot y^2 \cdot y^{\frac{1}{4}} \cdot y^{-\frac{2}{3}} = 3^{\frac{1}{2}+\frac{1}{2}} \cdot x^{\frac{1}{2}+\frac{1}{4}} \cdot y^{2+\frac{1}{4}-\frac{2}{3}} = 3x^{\frac{3}{4}} y^{\frac{19}{12}}$

17) $\frac{\sqrt[3]{2\sqrt{2}}}{\sqrt[6]{\sqrt{2}+1} \cdot \sqrt[6]{\sqrt{2}-1}} = \frac{\left(2 \cdot 2^{\frac{1}{2}} \right)^{\frac{1}{3}}}{\left(2^{\frac{1}{2}}+1 \right)^{\frac{1}{6}} \cdot \left(2^{\frac{1}{2}}-1 \right)^{\frac{1}{6}}} = \frac{\left(2^{1+\frac{1}{2}} \right)^{\frac{1}{3}}}{\left[\left(2^{\frac{1}{2}}+1 \right) \cdot \left(2^{\frac{1}{2}}-1 \right) \right]^{\frac{1}{6}}} = \frac{\left(2^{\frac{3}{2}} \right)^{\frac{1}{3}}}{\left[\left(2^{\frac{1}{\cancel{2}}} \right)^{\cancel{2}} - 1 \right]^{\frac{1}{6}}} = $

         $= \frac{2^{\frac{\cancel{3}}{2} \cdot \frac{1}{\cancel{3}}}}{\left[2^1 - 1 \right]^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{1^{\frac{1}{6}}} = \frac{2^{\frac{1}{2}}}{1} = 2^{\frac{1}{2}} = \sqrt{2}$

18) $\sqrt[13]{\left(\sqrt{t+1} \right)^3 \cdot \sqrt[3]{(t+1)^2}} = \left[(t+1)^{\frac{3}{2}} \cdot (t+1)^{\frac{2}{3}} \right]^{\frac{1}{13}} = \left[(t+1)^{\frac{3}{2}+\frac{2}{3}} \right]^{\frac{1}{13}} = \left[(t+1)^{\frac{\cancel{13}}{6}} \right]^{\frac{1}{\cancel{13}}} = (t+1)^{\frac{1}{6}} = \sqrt[6]{t+1}$