1)

$$\frac{1}{2}+\left(\frac{2x+1}{2x+4}-\frac{x^2+1}{x^2+4x+4}\right)\cdot\left(x+4+\frac{4}{x}\right)=$$

$$=\frac{1}{2}+\left[\frac{2x+1}{2(x+2)}-\frac{x^2+1}{(x+2)^2}\right]\cdot\frac{x^2+4x+4}{x}=$$

$$=\frac{1}{2}+\frac{(x+2)(2x+1)-2(x^2+1)}{2(x+2)^2}\cdot\frac{(x+2)^2}{x}=$$

$$=\frac{1}{2}+\frac{\cancel{2x^2}+x+4x+\cancel{2}-\cancel{2x^2}-\cancel{2}}{2\cancel{(x+2)^2}}\cdot\frac{\cancel{(x+2)^2}}{x}=$$

$$=\frac{1}{2}+\frac{5\cancel{x}}{2}\cdot\frac{1}{\cancel{x}}=\frac{1}{2}+\frac{5}{2}=\frac{6}{2}=3$$

2)

$$\left(\frac{1}{a^2+3a+2}-\frac{1}{a^2+4a+3}\right)\left(a^2+5a+6\right)=\left[\frac{1}{(a+1)(a+2)}-\frac{1}{(a+1)(a+3)}\right](a+2)(a+3)=$$

$$=\frac{\cancel{a}+3-\cancel{a}-2}{(a+1)\cancel{(a+2)}\cancel{(a+3)}}\cancel{(a+2)}\cancel{(a+3)}=\frac{1}{a+1}$$

3)

$$\left(\frac{a}{a^2-1}-\frac{a}{a^2+2a+1}\right)\cdot\frac{a^3+a^2-a-1}{a}=$$

$$=\left[\frac{a}{(a+1)(a-1)}-\frac{a}{(a+1)^2}\right]\cdot\frac{a^2(a+1)-(a+1)}{a}=$$

$$=\frac{a(a+1)-a(a-1)}{(a+1)^2(a-1)}\cdot\frac{(a+1)\left(a^2-1\right)}{a}=$$

$$=\frac{\cancel{a^2}+a-\cancel{a^2}+a}{\cancel{(a+1)^2}\cancel{(a-1)}}\cdot\frac{\cancel{(a+1)(a+1)(a-1)}}{a}=$$

$$=\frac{2\cancel{a}}{\cancel{a}}=2$$

4)

$$\left(\frac{x}{x+4}+\frac{x+4}{x}-2\right)\left(\frac{x+4}{4}-1\right)=\frac{x^2+(x+4)^2-2x(x+4)}{x(x+4)}\cdot\frac{x+\cancel{4}-\cancel{4}}{4}=$$

$$=\frac{\cancel{x^2}+\cancel{x^2}+\cancel{8x}+16-\cancel{2x^2}-\cancel{8x}}{x(x+4)}\cdot\frac{x}{4}=\frac{\overset{4}{\cancel{16}}}{\cancel{x}(x+4)}\cdot\frac{\cancel{x}}{\cancel{4}}=\frac{4}{x+4}$$

8)

$$\left(\frac{1}{y^2-4y}-\frac{8}{y^3-4y^2}+\frac{16}{y^4-4y^3}\right):\left(\frac{1}{y^2}-2\cdot\frac{2}{y^3}\right)=$$

$$=\left[\frac{1}{y(y-4)}-\frac{8}{y^2(y-4)}+\frac{16}{y^3(y-4)}\right]:\left(\frac{1}{y^2}-\frac{4}{y^3}\right)=\frac{y^2-8y+16}{\cancel{y^3}(y-4)}\cdot\frac{\cancel{y^3}}{y-4}=\frac{\cancel{(y-4)^2}}{\cancel{(y-4)^2}}=1$$

9)

$$\left(\frac{n-2}{3n^2-12n+9}-\frac{1}{6n-18}\right)(6n-6)=\left[\frac{n-2}{3\left(n^2-4n+3\right)}-\frac{1}{6(n-3)}\right]\cdot 6(n-1)=$$

$$=\left[\frac{n-2}{3(n-1)(n-3)}-\frac{1}{6(n-3)}\right]\cdot 6(n-1)=\frac{2n-4-n+1}{\cancel{6(n-1)}(n-3)}\cdot\cancel{6(n-1)}=\frac{\cancel{n-3}}{\cancel{n-3}}=1$$

16)

$$\left(\frac{1}{x^3-x^2-x+1}-\frac{1}{x^3+x^2-x-1}\right)\left(x^2-1\right)^2=$$

$$=\left[\frac{1}{x^2(x-1)-(x-1)}-\frac{1}{x^2(x+1)-(x+1)}\right](x+1)^2(x-1)^2=$$

$$=\left[\frac{1}{(x-1)\left(x^2-1\right)}-\frac{1}{(x+1)\left(x^2-1\right)}\right](x+1)^2(x-1)^2=$$

$$=\left[\frac{1}{(x-1)(x+1)(x-1)}-\frac{1}{(x+1)(x+1)(x-1)}\right](x+1)^2(x-1)^2=$$

$$=\left[\frac{1}{(x-1)^2(x+1)}-\frac{1}{(x+1)^2(x-1)}\right](x+1)^2(x-1)^2=\frac{\cancel{x}+1\cancel{-x}+1}{\cancel{(x-1)^2(x+1)^2}}\cancel{(x+1)^2(x-1)^2}=2$$

18)

$$\left(a\cdot\frac{5a+4}{4a^3-4a^2-a+1}-\frac{2a+1}{2a^2-3a+1}\right)\cdot\left(4a^2-1\right)=\left[\frac{a(5a+4)}{4a^2(a-1)-(a-1)}-\frac{2a+1}{2a^2-2a-a+1}\right]\cdot\left(4a^2-1\right)=$$

$$=\left[\frac{a(5a+4)}{(a-1)\left(4a^2-1\right)}-\frac{2a+1}{2a(a-1)-(a-1)}\right]\cdot\left(4a^2-1\right)=\left[\frac{a(5a+4)}{(a-1)(2a+1)(2a-1)}-\frac{2a+1}{(a-1)(2a-1)}\right]\cdot\left(4a^2-1\right)=$$

$$=\frac{a(5a+4)-(2a+1)^2}{(a-1)\cancel{(2a+1)(2a-1)}}\cdot\cancel{(2a+1)(2a-1)}=\frac{5a^2\cancel{+4a}-4a^2\cancel{-4a}-1}{a-1}=\frac{a^2-1}{a-1}=\frac{(a+1)\cancel{(a-1)}}{\cancel{a-1}}=a+1$$

22)

$$\left(\frac{1}{4a-8}-\frac{1}{6a-12}\right)\cdot\left(12a^2-12a-24\right)-a=$$

$$=\left[\frac{1}{4(a-2)}-\frac{1}{6(a-2)}\right]\cdot 12\left(a^2-a-2\right)-a=$$

$$=\frac{3-2}{\cancel{12}\,\cancel{(a-2)}}\cdot\cancel{12}\,(a+1)\,\cancel{(a-2)}-a=\cancel{a}+1\cancel{-a}=1$$

24)

$$\frac{\dfrac{a-1}{a+1}-\dfrac{a+1}{a-1}}{\dfrac{1}{a+1}-1}=\frac{\dfrac{(a-1)^2-(a+1)^2}{(a+1)(a-1)}}{\dfrac{\cancel{1}-a\cancel{-1}}{a+1}}=\frac{\cancel{a^2}-2a\cancel{+1}\cancel{-a^2}-2a\cancel{-1}}{\cancel{(a+1)}(a-1)}\cdot\frac{\cancel{a+1}}{-a}=\frac{-4\cancel{a}}{-\cancel{a}(a-1)}=\frac{4}{a-1}$$

26)

$$\frac{\frac{b^3}{b^6+b^4+b^2+1}-\frac{1}{b^3+b^2+b+1}}{b-1}\cdot\left(b^8-1\right)+1=$$

$$=\frac{\frac{b^3}{b^4\left(b^2+1\right)+\left(b^2+1\right)}-\frac{1}{b^2(b+1)+(b+1)}}{b-1}\cdot\left(b^4+1\right)\left(b^4-1\right)+1=$$

$$=\frac{\frac{b^3}{\left(b^2+1\right)\left(b^4+1\right)}-\frac{1}{(b+1)\left(b^2+1\right)}}{b-1}\cdot\left(b^4+1\right)\left(b^2+1\right)\left(b^2-1\right)+1=$$

$$=\frac{\frac{b^3(b+1)-\left(b^4+1\right)}{(b+1)\left(b^2+1\right)\left(b^4+1\right)}}{b-1}\cdot\left(b^4+1\right)\left(b^2+1\right)(b+1)(b-1)+1=$$

$$=\frac{\frac{\cancel{b^4}+b^3\cancel{-b^4}-1}{(b+1)\left(b^2+1\right)\left(b^4+1\right)}}{b-1}\cdot\left(b^4+1\right)\left(b^2+1\right)(b+1)(b-1)+1=$$

$$=\frac{b^3-1}{\cancel{(b+1)}\cancel{\left(b^2+1\right)}\cancel{\left(b^4+1\right)}}\cdot\frac{1}{\cancel{b-1}}\cdot\cancel{\left(b^4+1\right)}\cancel{\left(b^2+1\right)}\cancel{(b+1)}\cancel{(b-1)}+1=b^3\cancel{-1}\cancel{+1}=b^3$$

38)

$$\frac{1}{\left(\frac{x}{x^2-3xy+2y^2}+\frac{1}{2y-x}\right)\left(\frac{x}{y}-2\right)}+y=\frac{1}{\left[\frac{x}{(x-y)(x-2y)}-\frac{1}{x-2y}\right]\cdot\frac{x-2y}{y}}+y=$$

$$=\frac{1}{\frac{\cancel{x}\cancel{-x}+y}{(x-y)\cancel{(x-2y)}}\cdot\frac{\cancel{x-2y}}{y}}+y=\frac{1}{\frac{\cancel{y}}{x-y}\cdot\frac{1}{\cancel{y}}}+y=x\cancel{-y}\cancel{+y}=x$$

42)

$$3\left(\frac{x}{3x-6}-\frac{x}{x^2-x-2}\right)\cdot\left(x^2+x^1+x^0+x^{-1}\right)-x^2=3\left[\frac{x}{3(x-2)}-\frac{x}{(x+1)(x-2)}\right]\cdot\left(x^2+x+1+\frac{1}{x}\right)-x^2=$$

$$=\cancel{3}\cdot\frac{x^2+x-3x}{\cancel{3}(x+1)(x-2)}\cdot\frac{x^3+x^2+x+1}{x}-x^2=\frac{x^2-2x}{(x+1)(x-2)}\cdot\frac{x^2(x+1)+(x+1)}{x}-x^2=$$

$$=\frac{\cancel{x}\cancel{(x-2)}}{\cancel{(x+1)}\cancel{(x-2)}}\cdot\frac{\cancel{(x+1)}\left(x^2+1\right)}{\cancel{x}}-x^2=\cancel{x^2}+1\cancel{-x^2}=1$$

43)

$$\left[\frac{1}{4}\left(\frac{1}{x+4}-\frac{1}{x-4}\right)\right]^2\left(\frac{x^2+8x+16}{2}\right)^2=\left[\frac{1}{4}\frac{\cancel{x}-4\cancel{-x}-4}{(x+4)(x-4)}\right]^2\left[\frac{(x+4)^2}{2}\right]^2=$$

$$=\left[\frac{1}{\cancel{4}}\frac{-\cancel{8}^{2}}{(x+4)(x-4)}\right]^2\frac{(x+4)^4}{4}=\frac{\cancel{4}}{\cancel{(x+4)^2}(x-4)^2}\frac{(x+4)^{\cancel{4}^{2}}}{\cancel{4}}=\frac{(x+4)^2}{(x-4)^2}$$

44)

$$\left(\frac{1}{a^2-2a+1}-\frac{1}{a^2+2a+1}\right)\cdot\frac{a^4-2a^2+1}{4}=\left[\frac{1}{(a-1)^2}-\frac{1}{(a+1)^2}\right]\cdot\frac{(a^2-1)^2}{4}=$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)^2(a+1)^2}\cdot\frac{[(a+1)(a-1)]^2}{4}=$$

$$=\frac{a^2+2a+1-a^2+2a-1}{(a-1)^2(a+1)^2}\cdot\frac{(a+1)^2(a-1)^2}{4}=\frac{4a}{4}=a$$

50)

$$\left[6\left(\frac{\frac{1}{2}a}{a-b}+\frac{\frac{1}{3}a}{b-a}\right)+\frac{ab^3}{a^4-ab^3}\right]\cdot\left(a+b-\frac{ab}{a+b}\right)=$$

$$=\left[6\left(\frac{1}{2}a\cdot\frac{1}{a-b}+\frac{1}{3}a\cdot\frac{1}{b-a}\right)+\frac{ab^3}{a\left(a^3-b^3\right)}\right]\cdot\frac{(a+b)^2-ab}{a+b}=$$

$$=\left\{6\left[\frac{a}{2(a-b)}+\frac{a}{3(b-a)}\right]+\frac{b^3}{a^3-b^3}\right\}\cdot\frac{a^2+2ab+b^2-ab}{a+b}=$$

$$=\left\{6\left[\frac{a}{2(a-b)}-\frac{a}{3(a-b)}\right]+\frac{b^3}{a^3-b^3}\right\}\cdot\frac{a^2+ab+b^2}{a+b}=\left\{6\cdot\frac{3a-2a}{6(a-b)}+\frac{b^3}{a^3-b^3}\right\}\cdot\frac{a^2+ab+b^2}{a+b}=$$

$$=\left\{\frac{a}{a-b}+\frac{b^3}{(a-b)\left(a^2+ab+b^2\right)}\right\}\cdot\frac{a^2+ab+b^2}{a+b}=\frac{a^3+a^2b+ab^2+b^3}{(a-b)\left(a^2+ab+b^2\right)}\cdot\frac{a^2+ab+b^2}{a+b}=$$

$$=\frac{a^2(a+b)+b^2(a+b)}{(a-b)(a+b)}=\frac{(a+b)\left(a^2+b^2\right)}{(a-b)(a+b)}=\frac{a^2+b^2}{a-b}$$

52)

$$\frac{\frac{y+1}{y-1}+\frac{2}{(1-y)^2}+1}{1-y+y^2}=\frac{\frac{y+1}{y-1}+\frac{2}{(y-1)^2}+1}{1-y+y^2}=\frac{\frac{y^2-1+2+y^2-2y+1}{(y-1)^2}}{1-y+y^2}=$$

$$=\frac{2y^2-2y+2}{(y-1)^2}\cdot\frac{1}{1-y+y^2}=\frac{2\left(y^2-y+1\right)}{(y-1)^2}\cdot\frac{1}{\left(1-y+y^2\right)}=\frac{2}{(y-1)^2}$$

55)

$$1+\left(\frac{1}{n-1}-\frac{n}{n^2-1}\right)\left(n^4-2n^2+1\right)=1+\left[\frac{1}{n-1}-\frac{n}{(n+1)(n-1)}\right]\left(n^2-1\right)^2=$$

$$=1+\frac{n+1-n}{(n+1)(n-1)}\cdot(n+1)^2(n-1)^2=1+n^2-1=n^2$$

56)

$$1+\left[\frac{1}{(n-1)(n^2+n+1)}-\frac{n}{n^4-1}\right]\left[n^3+(n^2+1)^2+n\right]=$$

$$=1+\left[\frac{1}{(n-1)(n^2+n+1)}-\frac{n}{(n^2+1)(n+1)(n-1)}\right]\cdot\left[n^3+n+(n^2+1)^2\right]=$$

$$=1+\frac{(n+1)(n^2+1)-n(n^2+n+1)}{(n+1)(n-1)(n^2+n+1)(n^2+1)}\cdot\left[n(n^2+1)+(n^2+1)^2\right]=$$

$$=1+\frac{\cancel{n^3}\cancel{+n}\cancel{+n^2}+1\cancel{-n^3}\cancel{-n^2}\cancel{-n}}{(n+1)(n-1)\cancel{(n^2+n+1)}\cancel{(n^2+1)}}\cdot\cancel{(n^2+1)}\cancel{(n^2+n+1)}=$$

$$=1+\frac{1}{(n+1)(n-1)}=\frac{n^2\cancel{-1}\cancel{+1}}{(n+1)(n-1)}=\frac{n^2}{(n+1)(n-1)}$$

57)

$$\left(\frac{1}{a^2-2ab-3b^2}+\frac{1}{2b^2+ab-a^2}\right)\cdot\left(6ab^2-5a^2b+a^3\right)=$$

$$=\left[\frac{1}{(a-3b)(a+b)}-\frac{1}{a^2-ab-2b^2}\right]\cdot\left(a^3-5a^2b+6ab^2\right)=$$

$$=\left[\frac{1}{(a-3b)(a+b)}-\frac{1}{(a-2b)(a+b)}\right]\cdot a\left(a^2-5ab+6b^2\right)=$$

$$=\frac{\cancel{a}-2b\cancel{-a}+3b}{\cancel{(a-3b)(a-2b)}(a+b)}\cdot a\,\cancel{(a-2b)(a-3b)}=\frac{b}{a+b}\cdot a=\frac{ab}{a+b}$$

65)

$$\left(\frac{x+2}{x^3-8}-\frac{1}{x^2+2x+4}\right)\left(\frac{x}{4}-\frac{1}{2}\right)=\left[\frac{x+2}{(x-2)(x^2+2x+4)}-\frac{1}{x^2+2x+4}\right]\cdot\frac{x-2}{4}=$$

$$=\frac{\cancel{x}+2\cancel{-x}+2}{\cancel{(x-2)}(x^2+2x+4)}\cdot\frac{\cancel{x-2}}{4}=\frac{1}{x^2+2x+4}$$

72)

$$\frac{a^4+2a^3-2a-1}{a^3-3a-2}-1\underset{\text{RUFFINI}}{=}\frac{(a+1)^{\cancel{3}}(a-1)}{(a-2)\cancel{(a+1)^2}}-1=\frac{a^2-1-a+2}{a-2}=\frac{a^2-a+1}{a-2}$$

73)

$$\frac{a^4+a^3+4a}{a^4+4a^3+4a^2}=\frac{\cancel{a}\left(a^3+a^2+4\right)}{a^{\cancel{2}}\left(a^2+4a+4\right)}\underset{\text{RUFFINI}}{=}\frac{\cancel{(a+2)}\left(a^2-a+2\right)}{a(a+2)^{\cancel{2}}}=\frac{a^2-a+2}{a(a+2)}$$

74)

$$\left(\frac{b}{a^2-b^2-2b-1}+\frac{1}{a^2+ab+a}\right)\left(\frac{a^2}{b+1}-a\right)=$$

$$=\left[\frac{b}{a^2-(b+1)^2}+\frac{1}{a(a+b+1)}\right]\cdot\frac{a^2-a(b+1)}{b+1}=$$

$$=\left[\frac{b}{(a+b+1)(a-b-1)}+\frac{1}{a(a+b+1)}\right]\cdot\frac{a^2-ab-a}{b+1}=$$

$$=\frac{ab+a-b-1}{\cancel{a}\,(a+b+1)\,\cancel{(a-b-1)}}\cdot\frac{\cancel{a}\,\cancel{(a-b-1)}}{b+1}=$$

$$=\frac{a(b+1)-(b+1)}{a+b+1}\cdot\frac{1}{b+1}=$$

$$=\frac{\cancel{(b+1)}\,(a-1)}{a+b+1}\cdot\frac{1}{\cancel{b+1}}=\frac{a-1}{a+b+1}$$

76)

$$\left(x\cdot\frac{x^3-5x^2+11x-10}{2x^2-3x-2}-1\right)\cdot\left(-2-\frac{3}{x-1}\right)^2-2x^2=$$

$$=\left(\frac{x^4-5x^3+11x^2-10x-2x^2+3x+2}{2x^2-3x-2}\right)\cdot\left(\frac{-2x+2-3}{x-1}\right)^2-2x^2=$$

$$=\left[\frac{x^4-5x^3+9x^2-7x+2}{2x^2-3x-2}\right]\cdot\left(\frac{-2x-1}{x-1}\right)^2-2x^2\underset{\text{RUFFINI}}{=}\frac{(x-1)^3\,\cancel{(x-2)}}{\cancel{(x-2)}\,(2x+1)}\cdot\frac{(-2x-1)^2}{(x-1)^2}-2x^2=$$

$$=\frac{(x-1)^{\cancel{3}}}{\cancel{2x+1}}\cdot\frac{(2x+1)^{\cancel{2}}}{\cancel{(x-1)^2}}-2x^2=\cancel{2x^2}+x-2x-1\cancel{-2x^2}=-x-1=-(x+1)$$

80)

$$\left[\frac{b^4}{a(a-b)^2}-a\cdot\left(\frac{a}{b-a}\right)^2\right]:\left(a^3-a^{-1}b^4\right)+\frac{1}{(a-b)^2}=$$

$$=\left[\frac{b^4}{a(a-b)^2}-a\cdot\frac{a^2}{(b-a)^2}\right]:\left(a^3-\frac{1}{a}\cdot b^4\right)+\frac{1}{(a-b)^2}=$$

$$=\left[\frac{b^4}{a(a-b)^2}-\frac{a^3}{(a-b)^2}\right]:\left(a^3-\frac{b^4}{a}\right)+\frac{1}{(a-b)^2}=$$

$$=\frac{b^4-a^4}{a(a-b)^2}:\frac{a^4-b^4}{a}+\frac{1}{(a-b)^2}=$$

$$=\frac{\overset{-1}{\cancel{b^4-a^4}}}{\cancel{a}\,(a-b)^2}\cdot\frac{\cancel{a}}{\cancel{a^4-b^4}}+\frac{1}{(a-b)^2}=$$

$$=-\cancel{\frac{1}{(a-b)^2}}+\cancel{\frac{1}{(a-b)^2}}=0$$